Bayesian Analysis of a Dynamic Multivariate Spatial Ordered Probit Model

PING GAO Graduate School of Economics and Business Hokkaido University
HIKARU HASEGAWA Department of Economics Hokkaido University

1. INTRODUCTION

Following the dynamic ordered probit model with spatial dependences in Wang and Kockelman (2009) and models for multivariate ordinal outcomes in Jeliazkov et al. (2008), this paper proposed a dynamic multivariate spatial ordered probit (DMSOP) model, wherein the spatial dependencies are explained by using an additive error specification as in (Smith and LeSage, 2004). The DMSOP model is the first to capture temporal and spatial interactions simultaneously for the multivariate discrete ordered responses. In applying this model, the parameters are calculated using the Bayesian inference based on Markov chain Monte Carlo (MCMC) sampling.

2. MODEL SPECIFICATION

The observed response variable y_{ikst} is a censored form of the latent response variable z_{ikst}:

$$y_{ikst} = l_s \text{ if } \gamma_{s,l_s-1} < z_{ikst} \leq \gamma_{s,l_s}, \text{ for } s = 1, \ldots, S, l_s = 1, \ldots, L_s, \text{and } t = 0, 1, \ldots, T, \tag{1}$$

where i indexes spatial regions ($i = 1, \ldots, m$), k indexes individuals inside each region ($k = 1, \ldots, n_i$). s indexes the sequence of observed response variables, and t indexes time periods. l_s indexes the classifications of response variable y_{ikst}. In addition, the cutpoints γ_{s,l_s} are specified as: $-\infty = \gamma_{s,0} < \gamma_{s,1} = 0 < \gamma_{s,2} < \cdots < \gamma_{s,L_s-1} = 1 < \gamma_{s,L_s} = \infty$.

The latent response variable z_{ikst} in the DMSOP model is specified as follows:

$$\begin{cases} z_{ikst} = \sum_{g=1}^{S} \lambda_{sg} z_{ikg(t-1)} + \boldsymbol{x}'_{ikst}\boldsymbol{\beta}_s + \delta_{ist} + \xi_{ikst}, \text{ for } t = 1, \ldots, T, \\ z_{iks0} = \boldsymbol{x}'_{iks0}\boldsymbol{\beta}_{s0} + \delta_{is0} + \xi_{iks0}, \text{ for } t = 0, \end{cases} \tag{2}$$

$$\delta_{is} = \rho \sum_{j=1}^{m} w_{ij}\delta_{js} + \epsilon_{is}, \text{ for } i = 1, \ldots, m, \text{and } s = 1, \ldots, S, \tag{3}$$

where λ_{sg} is the temporal coefficient between z_{ikst} and $z_{ikg(t-1)}$. δ_{ist} and ξ_{ikst} represent regional effects and individualistic effects for the sth outcome in period t, respectively. ρ is the spatial coefficient, which reflects the degree of spatial influence. Weights w_{ij} reflect the closeness between regions i and j.

3. CONCLUSIONS

The validity and accuracy of the model is verified by the simulated dataset. The DMSOP model performs very well with the simulated data. It satisfactorily explains the temporal and spatial dependences and interaction of multivariate response variables. In addition, the study illustrated the model by applying it to the survey data from China Family Panel Studies (CFPS) of adults in China. The spatial coefficient ρ and temporal coefficient $\boldsymbol{\Lambda}$ are large, and thus the spatial and temporal dependences in the empirical study are critical.

BIBLIOGRAPHY

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