Modeling Financial Durations with Limit Order Book Information

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Abstract. It is a stylized fact that durations between executions in financial markets have intraday seasonality and autocorrelation. The Autoregressive Conditional Duration (ACD) model has been widely used to capture these characteristics. However, durations are also supposed to be affected by liquidity in the market. We propose a new ACD model that utilizes the limit order book information for reflecting the liquidity. In our empirical analysis, we applied the proposed ACD model to high-frequency stock price data in the Tokyo Stock Exchange and estimated it with an efficient Markov chain Monte Carlo method. We also conducted model comparison among different specifications of the proposed model.

Keywords: Financial Durations, ACD, Mixed-ACD, MCMC, IWLS proposals

1   Model

In the Mixed-ACD Model proposed by Brownlees and Vannucci (2010), the expected value of duration $\mathbb{E}\left(x_i|x_{i-1},\ldots,x_1\right) = \mu_i$ is represented as

$$\mu_i = \eta_t \phi_{ti} \psi_{ti}, \tag{1}$$

where η_t, ϕ_{ti}, and ψ_{ti} are random daily effect, intraday seasonality, and duration clustering, respectively. The three components are given by

$$\eta_t = \exp\left(u_t\right), \tag{2}$$

$$\phi_{ti} = \exp\left\{\sum_{j=1}^{k} \gamma_k B_j\left(\tau_{ti-1}\right)\right\}, \tag{3}$$

$$\psi_{ti} = \omega + \alpha\left(\frac{x_{ti-1}}{\eta_t \phi_{ti}}\right) + \beta\psi_{ti-1}, \tag{4}$$

where $u_t|u_{t-1} \sim N\left(\rho u_{t-1}, \sigma_u^2\right)$, and B is a B-spline linear basis expansion.

By incorporating limit order book information to (3), we can confirm whether the information has a significant effect on the stock trading durations.

References

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