Robust change point detection by self-weighted GEL method

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This talk considers the change point detection of an observed stretch $\{y_1, \dots, y_n\}$ generated from the autoregressive moving average (ARMA) model

$$y_t = \mu + b_1 y_{t-1} + \dots + b_p y_{t-p} + e_t + a_1 e_{t-1} + \dots + a_q e_{t-q},$$

where $\{e_t : t \in \mathbb{Z}\}$ is a sequence of i.i.d. random variables with zero median. Throughout this talk, finite variance of e_t is not required, and then we can consider the possibly infinite variance models. Suppose that the coefficient vector $\beta := (\mu, b_1, \dots, b_p, a_1, \dots, a_q)^\top$ satisfies

$$\beta = \beta_{t,n} = \begin{cases} \beta_1 & t = 1, \dots, k_{0n} \\ \beta_2 & t = k_{0n} + 1, \dots, n \end{cases},$$

where $\beta_1, \beta_2 \in \mathbb{R}^{p+q+1}$ and k_{0n} is the unknown change point. In this talk, the knowledge of change point location is not assumed. The change point hypothesis is then represented as

$$H : \beta_1 = \beta_2 \quad \text{v.s.} \quad A : \beta_1 \neq \beta_2. \tag{1}$$

To test the hypothesis (1), we define the GEL statistic

$$l_{n,k}(\theta_1, \theta_2) := \sup_{\lambda} \sum_{i=1}^{k} \rho(\lambda^\top g_i^*(\theta_1)) + \sup_{\lambda} \sum_{i=k+1}^{n} \rho(\lambda^\top g_i^*(\theta_2)), \tag{2}$$

where $g_i^*(\theta)$ is the self-weighted moment function for ARMA processes, which is found in Akashi (2017). The idea of self-weighting method for time series models was originally proposed by Ling (*JRSS*, 67(3):381-393, 2005), and generalized to the ARMA models by Pan et al. (*Econom. Theory*, 23:852–879, 2007). By setting $\rho(v) = \log(1 - v)$ and using Lagrange multiplier method, the statistic (2) is regarded as a natural extension of the empirical likelihood ratio statistic by Akashi, Dette & Liu (2018) to the change point problem (1). Under some appropriate conditions, the limit distribution of the test statistic is given as follows.

Theorem. Suppose that $k_{0n}/n \to u_0 \in (0,1)$. Then, under $H : \beta_1 = \beta_2$,

$$T_n := 2 \max_{k_{1n} \le k \le k_{2n}} \left[\inf_{\beta \in \mathcal{B}} l_{n,k}(\beta, \beta) \right] \xrightarrow{d} 2 \max_{r_1 \le r \le r_2} \left[\frac{\|B(r) - rB(1)\|^2}{r(1-r)} \right] \qquad (n \to \infty),$$

where $\{B(r) : r \in [0,1]\}$ is the $(p + q + 1)$-dimensional vector of independent Brownian motions, $k_{jn} = r_j n$, and $r_1, r_2 \in (0,1)$ $(r_1 < r_2)$ are some constants.

So, if the test statistic T_n is significantly large, the null hypothesis of no structural change is rejected. The finite sample performance of the proposed test is also shown.

Reference

[1] Akashi, F. (2017). Self-weighted generalized empirical likelihood methods for hypothesis testing in infinite variance ARMA models. *Statistical Inference for Stochastic Processes*. 20(3), pp.291-313.

[2] Akashi, F., Dette, H. & Liu, Y. (2018). Change point detection in autoregressive models with no moment assumptions. To appear in *Journal of Time Series Analysis*. DOI: 10.1111/jtsa.12405.