# Nonparametric Inference on Compound Poisson-Driven Ornstein-Uhlenbeck Processes

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## 1 Introduction

Given a positive number $\lambda$ and an increasing Lévy process $J = (J_t)_{t\geq 0}$ without drift component, an Ornstein-Uhlenbeck (OU) process $X = (X_t)_{t\geq 0}$ driven by $J$ is defined by a solution to the following stochastic differential equation

$$dX_t = -\lambda X_t dt + dJ_{\lambda t},\ t \geq 0. \tag{1}$$

We focus on the case that the Lévy process $J$ in (1) is a compound Poisson process, that is, $J$ is of the form $J_t = \sum_{j=1}^{N_t} U_j,\ t \geq 0$, where $N = (N_t)_{t\geq 0}$ is a Poisson process with intensity $\alpha > 0$ and $\{U_j\}_{j\geq 1}$ is a sequence of i.i.d. positive-valued random variables with common distribution $F$. Under some conditions, the unique stationary distribution $\pi$ of $X$ is self-decomposable with the characteristic function

$$\varphi(t) = \int_{\mathbb{R}} e^{itx}\pi(dx) = \exp\left(\int_0^\infty (e^{itx} - 1)\frac{k(x)}{x}dx\right),\ k(x) = \alpha F([x,\infty))1_{[0,\infty)}. \tag{2}$$

## 2 Nonparametric inference on Lévy measures

We propose a new spectral estimator for the Lévy measure ($k$-function) of the process $X$ under macroscopic observations, that is, we have discrete observations $X_\Delta, X_{2\Delta}, \ldots, X_{n\Delta}$ at frequency $1/\Delta > 0$ with $\Delta = \Delta_n \to \infty$ and $\Delta_n/n \to 0$ as $n \to \infty$. Under this set up, we derive multivariate central limit theorems for the estimator over a finite number of design points. We also derive high-dimensional central limit theorems for the estimator in the case that the number of design points increases as the sample size increases by using high-dimensional Gaussian approximation technique developed in Chernozhukov et al. (2013). Building upon these asymptotic results, we develop methods to construct confidence bands for the Lévy measure and propose a practical method for bandwidth selection. Our results can be seen as complemental results in Jongbloed et al. (2005) and Ilhe et al. (2015) for nonparametric inference on Lévy measures of Lévy-driven OU processes.

## References

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