Non-asymptotic Bayesian minimax adaptation in several nonparametric models

Keisuke Yano, The University of Tokyo
Fumiyasu Komaki, The University of Tokyo, RIKEN Brain Science Institute

1. Abstract

Performance bounds for nonparametric Bayesian procedures are presented. In particular, non-asymptotic minimax adaptation of Bayesian procedures is discussed in the following three models:

- mean estimation in nonparametric regression models;
- predictive density estimation in nonparametric regression models (Yano and Komaki, 2017);
- density estimation with infinite-dimensional exponential families.

If there is enough time, the results in larger models such as the three-layer perceptron model are also presented.

2. Definition of non-asymptotic minimax adaptation

In nonparametric statistics, various classes of smoothness and variation (radius) are considered. One typical example is the Sobolev class with smoothness α and variation B, that is, the class of functions that are α-times differentiable with the L^2 norms of the derivatives bounded above by B. Another example is the Besov class that includes the Hölder-type smoothness class.

When the classes of smoothness and variation are given, an estimator that achieves simultaneously minimaxity up to a constant independent of sample size and variation in each of those classes is said to be non-asymptotically minimax adaptive. Consider estimation of an infinite-dimensional parameter f in class $\mathcal{F}(\alpha, B)$ with smoothness α and variation B. We denote by $R(f, \hat{f})$ the risk of an estimator $\hat{f}$ when the true parameter value is f. In this set-up, an estimator $\hat{f}$ is said to be non-asymptotically minimax adaptive if for any $\alpha > 0$, there exists a positive constant C independent of both sample size n and variation B such that the inequality

$$\sup_{f \in \mathcal{F}(\alpha, B)} R(f, \hat{g}) \leq C \inf_{\hat{f}} \sup_{f \in \mathcal{F}(\alpha, B)} R(f, \hat{f})$$

holds uniformly in $B > 0$ and $n \geq 1$.

References

[1] K. Yano and F. Komaki, *Non-asymptotic Bayesian Minimax Adaptation in Continuous-time Gaussian Channel*, arXiv:1609.00940.

[2] K. Yano and F. Komaki (2017). *Asymptotically minimax prediction in infinite sequence models*, Electron. J. Statist., vol. 11, pp. 3165–3195.