Asymptotic mixed normality of realized covariance in high-dimensions

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Let us consider d assets traded on a financial market. We model the log-price process of those d assets by a d-dimensional continuous Itô semimartingale $Y = (Y_t)_{t\in[0,1]}$. We denote by Y^i the i-th component of Y for every $i = 1, \dots, d$. Suppose that we observe the process Y at the equidistant times $t_h = h/n$, $h = 0, 1, \dots, n$. The aim of this talk is to implement statistical inference for the quadratic covariation matrix $[Y,Y]_1 := ([Y^i, Y^j]_1)_{1\le i,j\le d}$ based on the observation data $(Y_{t_h})_{h=0}^n$ when n tends to infinity. In this situation, the most canonical estimator for $[Y,Y]_1$ would be the *realized covariance matrix*:

$$\widehat{[Y,Y]}_1^n := \sum_{h=1}^{n} (Y_{t_h} - Y_{t_{h-1}})(Y_{t_h} - Y_{t_{h-1}})^\top.$$

We achieve our aim once we derive (an approximation of) the distribution of the estimation error $\widehat{[Y,Y]}_1^n - [Y,Y]_1$. In the low-dimensional setting, i.e. if d does not depend on n, there is the celebrated theory by [3] which enables us to establish the stable convergence in law of the variables $\sqrt{n}\left(\widehat{[Y,Y]}_1^n - [Y,Y]_1\right)$ to a mixed normal distribution (see e.g. Theorem 5.4.2 of [4]). In contrast, in the high-dimensional setting, i.e. if d diverges as n tends to infinity, there seems no theory to approximate the distribution of $\widehat{[Y,Y]}_1^n - [Y,Y]_1$ in a suitable sense so far. In this talk, we fill in this gap by establishing a mixed-normal approximation for probabilities that (the vectorization of) $\sqrt{n}\left(\widehat{[Y,Y]}_1^n - [Y,Y]_1\right)$ belongs to random sparsely convex polytopes. The proof is based on a variant of the recent high-dimensional central limit theory for sums of independent random vectors developed by [1, 2], where we accommodate the theory to random asymptotic covariance matrices.

References

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