A semi-parametric spatiotemporal Hawkes-type point process model with periodic background for crime data

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Past studies have shown that crime events are often clustered (Mohler et al., 2011). This study proposes a spatiotemporal Hawkes-type point process model, which includes a background component with daily and weekly periodization, and a clustering component that is triggered by previous events. Its conditional intensity function has a form of

$$\begin{aligned}\lambda(t,x,y) &\equiv \lim_{\Delta_t,\Delta_x,\Delta_y\downarrow 0}\frac{1}{\Delta_t\Delta_x\Delta_y}\Pr\left\{N\left([t,t+\Delta)\times(x,x+\Delta_x)\times(y,y+\Delta_y)\right)=1\mid\mathcal{H}_t\right\}\\ &= \mu_0\,\mu_{\mathrm{t}}(t)\,\mu_{\mathrm{d}}(t)\,\mu_{\mathrm{w}}(t)\,\mu_{\mathrm{b}}(x,y)+A\int_{-\infty}^{t-}\iint_S g(t-s)\,h(x-u,y-v)\,N(\mathrm{d}u\times\mathrm{d}v\times\mathrm{d}s),\end{aligned}$$

where $\mathcal{H}_t$ denotes the σ-algebra generated by the observational history of the process N before time t but not including t, A and μ_0 are relaxation coefficients to be estimated, the average values of background trend $\mu_{\mathrm{t}}(t)$, daily periodicity $\mu_{\mathrm{d}}(t)$, weekly periodicity $\mu_{\mathrm{w}}(t)$ and spatial background rate $\mu_{\mathrm{b}}(x,y)$ are all standardized to 1, and the temporal response g and spatial response h are p.d.f.s.

We generalize the nonparametric stochastic reconstruction method developed by Zhuang et al. (2004) and Zhuang (2006) so that we can estimate each component in the background rate and the triggering response that appears in the model conditional intensity: the background rate includes a daily and a weekly periodicity, a separable spatial component, and a long-term background trend, and the triggering response. The new stochastic reconstruction method developed in this study fits better to crime data and is simple to understand and to estimate, without requiring much prior knowledge of the studied phenomena. Using this method, we verify the existence of periodic components and the triggered effect in the process of the studied crime phenomena. Another key point is that we introduce the relaxation coefficients to stabilize and fasten the estimation process. In the estimation procedures, these two relaxation parameters are estimated by MLE when the background components and the excitation response functions are given. We have developed an iterative algorithm to estimate the background and clustering components and the relaxation parameters simultaneously.

The above model and method are applied the data of the occurrences of violence or robbery related to crimes in the city of Castellon, Spain, during two years. The results show that, as a social phenomenon, this kind of crime is highly influenced by the daily life rhythms, revealed by its daily and weekly periodicity, and that about 3% of such crimes can be explained by clustering. Further diagnostic residual analysis show that the model could be improved by considering the following ingredients: (1) Background dominates the whole process while the clustering effect only contributes about 3%. (2) The periodicity effect is strong in the background. (3) Residual analysis shows that crime activity is different during weekends from working days. (4) Downtown has different characteristics in crime activities from suburb regions.

References

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