# Inequalities for minimax Rényi divergence

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## 1. Abstract

Rényi divergence is an important generalization of Kullback divergence in Information theory. It has an operational characterization in data compression and hypothesis testing. Rényi divergence of order 1 is Kullback divergence. Rényi divergence of order $\infty$ is the worst case regret of coding. In this talk, I show inequalities for minimax Rényi divergence. The inequalities ensure that distributions having maximum Rényi divergences close to the minimax Rényi divergence are close to the minimax distribution in Rényi divergence. Surprisingly, it is known that this natural inequality does not hold when $\alpha$=0. I show that this inequality holds for $\alpha \in (0, \infty]$ and present generalizations of this inequality to the cases in which observations are available at the beginning of prediction. Several techniques including Pythagorean inequalities and a characterization of minimax Rényi divergence are also introduced.

## 2. Definition of Rényi divergence

For two probability measures $P$ and $Q$ absolutely contiguous with respect to a common $\sigma$-finite measure $\mu$, the Rényi divergence $D_\alpha(P \mid\mid Q)$ of order $\alpha \in (0,1) \cup (1, \infty)$ is defined as

$$D_\alpha(P \mid\mid Q) = \frac{1}{\alpha - 1} \log \int p^\alpha q^{1-\alpha} d\mu,$$

where the conventions that $0/0 = 0$ and $x/0 = \infty$ for any $x > 0$ are adopted. The Rényi divergences of orders 0,1, and $\infty$ is defined by taking the limit; see Csiszár (1995) and van Erden and Harremoës (2014).

## References

[1] I. Csiszár, *Generalized cutoff rates and Rényi's information measures*, IEEE Transactions on Information Theory, vol. 41, pp. 26–34, 1995.

[2] T. van Erden and P. Harremoës, *Rényi Divergence and Kullback–Leibler Divergence*, IEEE Transactions on Information Theory, vol. 60, pp. 3797–3820, 2014.