Optimal Bayesian Minimax Rates for Unconstrained Large Covariance Matrices

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We obtain the optimal Bayesian minimax rate for the unconstrained large covariance matrix of multivariate normal sample with mean zero, when both the sample size, n, and the dimension, p, of the covariance matrix tend to infinity.

Traditionally the posterior convergence rate is used to compare the frequentist asymptotic performance of priors, but defining the optimality with it is elusive. We propose a new decision theoretic framework for prior selection and define *Bayesian minimax rate*.

Under the proposed framework, we obtain the optimal Bayesian minimax rate for the spectral norm for all rates of p. We also considered Frobenius norm, Bregman divergence and squared log-determinant loss and obtain the optimal Bayesian minimax rate under certain rate conditions on p. A simulation study is conducted to support the theoretical results.