# Particle rolling estimation with marginalized block sampling

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This report is concerned with rolling estimation for Bayesian inference in state space models (SSMs) by computational methodologies. SSMs consist of an observation vector $y_t$, an unobserved state vector $\alpha_t$ and a static parameter vector $\theta$, with prior density $p(\theta)$. In rolling estimation, one sequentially estimates a collection of posteriors with its data width fixed $L$, $p(\theta, \alpha_{t-L+1:t} \mid y_{t-L+1:t})$ for $t = L+1, \ldots, T$, where $z_{k:l}$ denotes a vector $(z_k, \ldots, z_j)'$. This type estimation is implemented to observe change of structure which generates time series data for financial time series data, econometric time series data, and so on. Estimation of a lot of posteriors, which rolling window estimation requires, is possible when each posterior takes tractable form but otherwise this estimation is not realistic in terms of computational cost. That is, simply applying Markov chain Monte Carlo (MCMC) methodologies, which are often used when the model is complicated, to each estimation takes huge amount of time. This is because only one MCMC sampling for drawing the posterior distribution can spend several hours. Then we propose a new computational method to estimate these posterior in the framework of sequential Monte Carlo (SMC) methods, in which a set of weighted particles approximating each posterior is updated sequentially. In the new algorithm, incorporating new information and discarding old information is done with a novel tool named marginalized block sampling, which is a combination of the existing block sampling technique and the particle MCMC algorithm. We show this new algorithm is justified in the framework of SMC by using the idea of the particle MCMC methodology. In addition, we apply the new method to S&P500 data analysis using the realized stochastic volatility with leverage model.

## References

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