On a mode preserving circular distribution and its Bayesian inference

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Abstract

In this study, we consider unimodal skew circular distributions through inverse monotone functions. General properties of the distributions together with skewness measures based on the distribution and density functions are provided. The inverse k-sine-skewed circular distributions are introduced as special cases of this type. More flat-topped and sharply peaked version of the distributions are also given. General results are also provided for maximum likelihood estimation (MLE) of the parameters and Fisher information matrix of the distributions. To calculate Bayes estimates for the model parameters, we introduce the importance sampling estimation algorithms. We also provide approximate Bayes estimates using Lindley's approximation based on both MLE and MAP estimates. Monte Carlo simulations are performed to compare the performance of the Bayes estimates with the classical estimates. Three circular datasets are analyzed for illustrative purposes.

keywords : Bayesian estimation, circular statistics, importance sampling, k-sine-skewed circular distributions, likelihood-based inference, skewness measures, unimodality

1 Introduction

Let $f_0(\theta)$ and $g(\theta)$ ($\theta \in [-\pi, \pi)$) be circular pdf's symmetric about $\theta = 0$ and $G(\theta) = \int_{-\pi}^{\theta} g(\phi)d\phi$ is the distribution function of the latter. The weighting function w is an odd periodic function with $|w(\theta)| \leq \pi$. Then

$$f(\theta) = 2f_0(\theta)G\{w(\theta)\}, \quad -\pi \leq \theta < \pi$$

is a circular pdf, see more details Umbach and Jammalamadaka (SPL, 79, 659–663, 2009). On the other hand, Jones (Stat. Sinica, 24, pp. 749–771, 2014) proposed a class of skew distributions with pdf

$$f(x) = f_0(s^{-1}(x)), \quad x \in S_f,$$

where $s'(x)+s'(-x) = 2$, the function s is a monotone and bijection function and S_f is a support of the function s.

Combining these previously proposed circular densities, we consider the *unimodal* skew distributions on the circle given by

$$f(\theta) = f_0(s^{-1}(\theta; w)), \quad \theta \in [-\pi, \pi),$$

which is an extension of the model in Jones and Pewsey (JASA, 100, pp.1422-1428, 2012), where f_0 is a symmetric unimodal density about $\theta = 0$ and the function s is given by

$$s(\theta; w) = 2\int_{-\pi}^{\theta} G(w(t))dt - \pi.$$

In this study, the parameter estimation based on the maximum likelihood and Bayesian methods are discussed.