

Quasi likelihood analysis and asymptotic theory of sparse estimation for stochastic processes

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The quasi likelihood analysis (QLA) gives a basis of statistical inference for stochastic processes. The polynomial type large deviation (PLD) inequality featuring in the QLA provides estimates of the tail of the quasi likelihood random field and hence L^p -boundedness of the quasi maximum likelihood estimator and the quasi Bayesian estimator. The PLD inequality follows from the local asymptotic quadratic structure of the quasi log likelihood function ([12]). Since it does not use any specific structure of stochastic models, the theory of the QLA can apply to various stochastic processes: diffusion processes ([12], [7], [9], [8]), jump-diffusion processes ([3]), non-synchronous sampling ([4]), model selection ([6],[10]) and point processes applied to limit order books ([1], [2], [5]). Recently the QLA is used in sparse estimation ([11]).

We formulate penalized methods within the QLA and show that the ordinary QLA without penalty entails a penalized QLA, keeping PLD. Consequently, we obtain L^p -boundedness of the penalized estimator and a precise estimate of the probability of selection consistency. Similarly, we can formulate the least square approximation method in a general setting.

Since QLA without penalty has been well established, the penalized QLA so obtained can apply with high universality to various dependent structures. We will discuss applications of the QLA to the penalized methods for estimation of point processes and diffusion type processes.

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