The properties of estimators for line segment process

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1 Introduction

In this article we are developing a method to estimate of the parameters of line segment process. The line segments have been observed through the specified window. Then, the observed line segments could be non-censored, right-censored, left-censored or doubly-censored. Even though the distribution of lengths of line segments and numbers of line segments is independent, the distribution of the number of line segments is affected by the distribution of the length of the line segment. The properties of estimators will be investigated by simulation study.

Some spatial data can be modeled as line segments. Svensson *et al.* (2006) have used models of line segments to represent the fibers in the trunk of tree. Laslett (1982) proposed a non-parametric maximum likelihood estimator for the length distribution of the line segments. These studies deals with estimation of the length of line segments observed within the window. In some applications, we can obtain only partial observations through the limited window. These type of censoring are called "window censored" in the statistical literature (Abe and Kamakura, 2016; Zhu *et al.*, 2014; Rootzén & Zholud, 2016).

2 Model

We assume the midpoints of each line segments should be given by the law of a Poisson process of rate λ. The line segments have random length L_i and $P(L_i \leq l) = F(l)$ say. The $F(l)$ is a continuous distribution supported in $[0, \infty)$ and which has finite mean μ. Such line segments have been observed in the interval $[T_0, T_0 + w]$ $(w > 0)$. Only in the interval, we can observe the line segments. Let S_i be the starting point of the line segments in the window $[T_0, T_0 + w]$, and let E_i be the end of the observed part of line segments. Under these assumptions, We can derive the likelihood function for datasets from a window censored line segment process.

A simple and important special case is when F is an exponential distribution, $1-F(l) = e^{-l/\mu}$. Then the residual length distribution is same as the original distribution. In this case, the maximum likelihood estimator has a closed form. We can derive the asymptotic variance.

Reference

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