Error Reduction for Kernel Distribution Function Estimators

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In this talk we discuss the kernel type estimation of a distribution function. A method to reduce the mean integrated squared error for kernel distribution function estimators is proposed.

Let $X_1, X_2, ..., X_n$ be independently and identically distributed random variables with an absolutely continuous distribution function F_X and a density f_X. Nadaraya (1964) proposed the standard kernel distribution function esrimator as

$$\widehat{F}_h(x) = \frac{1}{n}\sum_{i=1}^{n} W\left(\frac{x - X_i}{h}\right), \quad x \in \mathbb{R}, \tag{1}$$

where $W(v) = \int_{-\infty}^{v} K(w)\mathrm{d}w$, K is a function called as kernel, and $h > 0$ is called as bandwidth, which is a smoothing parameter and controls the smoothness of $\widehat{F}_h$. It is usually assumed that K is a symmetric (about 0) continuous nonnegative function with $\int_{-\infty}^{\infty} K(v)\mathrm{d}v = 1$, as well as $h \to 0$ and $nh \to \infty$ when $n \to \infty$. Nadaraya (1964) derived some asymptotic properties of $\widehat{F}_h(x)$, and he got that the bias is in the order of $O(h^2)$ and the variance is in the order of $O\left(\frac{1}{n}\right)$.

Our purpose is to reduce the bias and the variance of the standard kernel distribution function estimator by using geometric extrapolation. The idea of this method is using a self-elimination technique between two standard kernel distribution function estimators with different bandwidths. We also use exponential and logarithmic expansions. Our proposed estimator is

$$\tilde{F}_X(x) = [\widehat{F}_h(x)]^{\frac{a^2}{a^2-1}}[\widehat{F}_{ah}(x)]^{-\frac{1}{a^2-1}}, \tag{2}$$

where $a (\neq 1)$ is a positive number, which acts as the second smoothing parameter. With some additional assumptions, it can be proved that the bias of $\tilde{F}_X(x)$ is in the order of $O(h^4)$, which means it is considerably smaller in the sense of convergence rate than that of the standard kernel distribution function estimator. Even though the rate of convergence of variance does not change, the variance of our proposed estimator is smaller up to some constants. As a result, the mean squared error can be reduced.

References

[1] Nadaraya, E. A. (1964) Some new estimates for distribution functions. *Theory Probability and Applications* Vol. 15, 497-500.

[2] Terrel, G. R. and Scott, D. W. (1980) On improving convergence rates for nonnegative kernel density estimation. *The Annals of Statistics* Vol. 8, 1160-1163.

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