Projection Smoothing for Stochastic Dynamical Systems

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In this study, we consider a problem of estimating the state of nonlinear stochastic dynamical systems from noisy observations. This problem arises in many application areas such as target tracking, navigation and data assimilations. The estimation problem is solved by computing the conditional probability of the state, conditioned on a sequence of observations. Given the observations up to time t, the problem is called *filtering* if we compute the conditional probability at time t; it is called *smoothing* if the conditional probability at any time before t is to be computed [1].

The filtering and smoothing problems can be solved by the forward and backward recursive-Bayesian algorithms, respectively. These algorithms involve solving partial differential equations (PDEs) the conditional probability densities satisfy. For linear Gaussian systems, the first two moments are sufficient for characterizing the conditional densities. Thus the PDEs are reduced to finite-dimensional ordinary differential equations (ODEs), i.e., the Kalman filter and smoother. For general nonlinear systems, however, the conditional probability densities cannot fully be characterized by finite number of moments, thereby the PDEs must either be numerically solved (including Monte Carlo approaches), or be approximated by finite dimensional ODEs; we will develop the later approach.

Here, we apply the projection method to the backward algorithm, yielding a novel finite dimensional approximation of nonlinear smoother which we call the *projection smoother*. Combining with the projection filter developed in [2], we formulate a finite dimensional approximation for the forward and backward algorithms, based on the projection method. We present a numerical study for a particular case of double-well potential system, and compare in estimation performance between our method and other methods based on Gaussian approximations that include the extended Kalman scheme.

[1] A. H. Jazwinski (1970) Stochastic Process and Filtering Theory, Academic Press.
[2] D. Brigo, B. Hanzon and F. L. Gland (1999) Approximate filtering by projection on the manifold of exponential densities, Bernoulli **5**, 495–534.