The Simultaneous Multivariate Hawkes-type Point Processes and their application to Financial Markets

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1. In economic and financial time series we sometimes observe large jumps. Although they are relatively rare events, they would have significant influence not only on a financial market but also several different markets and macro economies. By using the simultaneous Hawkes-type (SMHP) pmodels, which are new multivariate point processes, it is possible to analyze the causal effects of large events in the sense of the Granger-non-causality (GNC) and the instantaneous Granger-non-causality (IGNC). We investigate the financial market of Tokyo and other markets, and apply the Granger non-causality tests. We have found several important empirical findings among financial markets and macro economies.

2. For p-dimensional price processes $P_j(t)$ $(j = 1, \cdots, p\ ; t_{i-1}^n < t \leq t_i^n, i = 1, \cdots, n)$ in $s \in I_i$ the log-returns are defined by $Y_j^n(s) = -\log[P_j(s)/P_j(t_{i-1}^n)]$ $(j = 1, \cdots, p; i = 1, \cdots, n)$. Let the first stopping times over the threshold) u_j in $s \in I_i$ be $\tau_j^n(i,1)$, and for $s \in t_{i-1}^n < s \leq \tau_j^n(i,1)$ we define $X_j^n(s) = Y_j^n(s)$ for $s \in \tau_i^n(i,1) < s \leq t_i^n$. The point processes $N_j^n(s, u_j)$ are the number of exceedances of $X_j^n(s)$ over u_j $(j = 1, \cdots, p)$ and we use the self-exciting Hawkes-type intensity functions

$$\lambda_j^n(t,u) = \lambda_{j0} + \lambda_j \int_{-\infty}^{t} \sum_{i=1}^{p} c_{ji}(X_i^n(s-))g_i(t-s)dN_i^n(s,u)\ . \tag{1}$$

We also develop the SMHP models which incorporate simultaneous jumps and apply our models to investigate the international financial markets.

3.**References**

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