Modeling time scale for stochastic process [†]

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We are concerned here with how to quantify time scale based on discrete-time sample

$$(t_1, X_{t_1}),\ (t_2, X_{t_2}), \ldots, (t_n, X_{t_n})$$

from the parametric ergodic diffusion model $X = (X_t)_{t \in \mathbb{R}_+}$:

$$dX_t = a(X_t, \alpha) dw_t + b(X_t, \beta) dt.$$

We will focus on the equidistant and rapidly increasing experimental sampling condition: $h = h_n = t_j - t_{j-1}$ for all $j \le n$ and $nh^2 \to 0$, while $nh \to \infty$ as $n \to \infty$.

- On the one hand, it is well-known in the literature that under the standard smoothness conditions on the coefficients, the Gaussian quasi-likelihood estimator

$$\left(\hat{\alpha}_n(h),\ \hat{\beta}_n(h)\right) \in \operatorname{argmax} \sum_{j=1}^{n} \log \phi \left(X_{t_j};\ X_{t_{j-1}} + hb(X_{t_{j-1}}, \beta),\ ha^{\otimes 2}(X_{t_{j-1}}, \alpha) \right)$$

  theoretically efficiently works, where $\phi(\cdot; \mu, \Sigma)$ denotes the $N_d(\mu, \Sigma)$-density.

- On the other hand, however, one would get confused with the *practical* problem "what value is to be assigned to h", for in real data set the jth time stump t_j may be something like "`2017/6/26 15:50:45`" and there is no absolute correspondence between model time scale: that is to say, h is a fine-tuning parameter which does affect estimation results. One way is to subjectively endow h with a sufficiently small value, in connection with the terminal sampling time nh to be large enough.

When and how can we sidestep the subjective choice of h? Motivated by this question, in this talk we will propose a modified Gaussian quasi-likelihood function which is completely free from the fine-tuning of h and also leads to the following properties under an additional seemingly non-standard identifiability condition on the diffusion coefficient:

(1) The associated estimator is rate-efficient and asymptotically normally distributed;

(2) The sampling stepsize h can be estimated in some sense.

Also discussed will be possible model extensions including non-ergodic continuous semimartingale and Lévy driven SDE.

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