Asymptotic theory of parametric inference for ruin probability under Lévy insurance risks

早稲田大学・理工学術院　清水 泰隆
(Yasutaka SHIMIZU, Waseda University)

1. Ruin probability for Lévy surplus

The classical insurance ruin theory and its related fields can revive interest in recent Enterprise Risk Management (ERM) because the theory gives us many tools for the dynamic risk management. A central issue in this context is estimating ruin probability under certain spectrally negative jump processes.

Consider the following *Lévy insurance surplus process*:

$$R_t^\theta = u + ct - S_t^\alpha + \sigma W_t, \quad t \geq 0,$$

where $u > 0$ is the initial surplus, $c > 0$ is the (expected) premium rate, W is a Wiener process, and S^α is a *Lévy subordinator* (a Lévy process with increasing paths) with the Lévy measure ν_α, that is, the Laplace transform of S_t^α is given by

$$\mathbb{E}\left[e^{-uS_t^\alpha}\right] = \exp\left(t\int_0^\infty (1 - e^{-uz})\,\nu_\alpha(\mathrm{d}z)\right), \quad u \geq 0.$$

Put $\theta = (\alpha, D)$ with $D = \sigma^2/2$, where $\sigma \geq 0$ and $\alpha \in \Xi \subset \mathbb{R}^p$ are unknown parameters, and say θ_0 the true value of the parameter. The ruin probability is defined as

$$\psi_\theta(u) = \mathbb{P}\left(\inf_{t>0} R_t^\theta < 0 \,\middle|\, R_0^\theta = u\right).$$

2. A problem for asymptotic distribution

Suppose that we can construct an asymptotic normal estimator of θ from samples of $[0,T]$-surplus: for a sequence $r_T \to \infty$,

$$r_T \cdot (\widehat{\theta}_T - \theta_0) \xrightarrow{\mathcal{D}} N(0, \Sigma_{\theta_0}^2), \quad T \to \infty,$$

Then we obtain an asymptotically normal estimator of the ruin probability via the *delta method*:

$$r_T \cdot \left(\psi_{\widehat{\theta}_T}(u) - \psi_{\theta_0}(u)\right) \xrightarrow{\mathcal{D}} N\left(0, |\dot{\psi}_{\theta_0}(u)|^2\Sigma_{\theta_0}^2\right), \quad T \to \infty.$$

where $\dot{\psi}_\theta = \partial\psi_\theta/\partial\theta$. However, computing or estimating the derivative $\dot{\psi}_\theta$ is not so easy, which is inconvenient for constructing a confidence interval or testing hypothesis for ψ_{θ_0}.

3. Main results

[Cramér-type approximation for $\dot{\psi}_\theta$]

$$\dot{\psi}_\theta(u) \sim D_\theta u e^{-\gamma_\theta u}, \quad u \to \infty,$$

where $D_\theta > 0$ is a constant and $\gamma_\theta > 0$ is the *adjustment coefficient*.

[**Asymptotic** $(1-\alpha)$**-confidence interval**] There exists a random interval I_T^α s.t.

$$\lim_{T\to\infty} \mathbb{P}\left(\psi_{\theta_0}(u_T) \in I_T^\alpha\right) = 1 - \alpha$$

for a suitable sequence $u_T \to \infty$.

The details for regularity conditions, the constant D_θ, the concrete form of I_T^α, and also numerical experiments are presented in the talk.