Asymptotic properties and robustness for the Bayes estimator based on the divergence

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Abstract

Robust estimation based on the divergence measures have been developed in recent years. Basu et al. (1998) proposed robust estimators based on the density-power divergence and showed some asymptotic properties and robustness. On the other hand, Fujisawa and Eguchi (2008) introduced a robust estimator which is effective for heavily contaminated data based on the another type of divergence (called γ-divergence). It was shown that the robust estimator based on the γ-divergence has small latent bias without assuming that the contamination ratio is small.

In Bayesian contexts, Ghosh and Basu (2016) consider the robust Bayes estimation using the weighted likelihood function based on the density-power divergence. They also showed the asymptotic property of the Bayes estimator based on the density-power divergence, and characterized the robustness in terms of the influence function. However, the estimator based on the density-power divergence may not work well for estimation of the variance parameter and heavily contaminated data. In this work, we propose the posterior distribution based on the γ-divergence given by

$$\pi^{(\gamma)}(\boldsymbol{\theta}|\boldsymbol{X}_n) = \frac{\exp\{nQ_n^{(\gamma)}(\boldsymbol{\theta})\}\pi(\boldsymbol{\theta})}{\int \exp\{nQ_n^{(\gamma)}(\boldsymbol{\theta})\}\pi(\boldsymbol{\theta})d\boldsymbol{\theta}} \tag{1}$$

for $\gamma > 0$, where $\pi(\boldsymbol{\theta})$ is the prior density and $Q_n^{(\gamma)}(\boldsymbol{\theta})$ is the monotone transformed γ-cross entropy

$$Q_n^{(\gamma)}(\boldsymbol{\theta}) = \frac{1}{n}\sum_{i=1}^{n} \frac{\frac{1}{\gamma} f_{\boldsymbol{\theta}}(X_i)^{\gamma}}{\{\int f_{\boldsymbol{\theta}}(x)^{1+\gamma}dx\}^{\gamma/(1+\gamma)}} - \frac{1}{\gamma}.$$

When $\gamma \to 0$, $Q_n^{(\gamma)}(\boldsymbol{\theta})$ is the ordinary log-likelihood function. Therefore, $Q_n^{(\gamma)}(\boldsymbol{\theta})$ may be regarded as the generalization of the log-likelihood function. It is shown that the maximum a posteriori (MAP) estimator and posterior mean based on (1) have the asymptotic normality. Further, we consider the comparison of the influence functions for the posterior means of the mean and variance parameters between the posteriors based on the density-power and the γ-divergence through some simulation studies.

References

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