Efficient MCMC in high-dimension and yuima package

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Markov chain Monte Carlo algorithms are widely used methodologies for evaluating complicated integrals in Bayesian statistics. Recently, there has been an increased interest in evolutive Bayesian methodologies in the context of big data. Traditional Markov chain Monte Carlo algorithms suffer from dimensionality. For example, the computational cost of the random-walk Metropolis algorithm in d-dimension is not a linear order in d but d^2 for an ideal setting, and d^3 for a heavy-tail setting. High-dimensional asymptotic theory is a useful tool for this type of analysis.

High-dimensional asymptotic theory is at least dates back to [1] and has been formalized and developed by [3]. Here, we illustrate a simple example of this "curse of dimensionality". Suppose that the random-walk Metropolis updates the current value X_m^d with proposal

$$X_{m+1}^{d*} = X_m^d + \sigma_d W_m^d,\ W_m^d \sim N_d(0, I_d)$$

where $\sigma_d > 0$ is a tuning parameter and $\{X_m^d\}_m$ is the Markov chain generated by this algorithm. Under stationarity assumption, the law of (X_m^d, X_{m+1}^d) and (X_{m+1}^d, X_m^d) are the same. Thus, the acceptance probability is

$$\begin{aligned}\mathbb{P}(X_1^d \neq X_0^d) &= \mathbb{P}(\|X_1^d\|^2 \neq \|X_0^d\|^2) = 2\mathbb{P}(\|X_1^d\|^2 < \|X_0^d\|^2) \\ &\leq 2\mathbb{P}(\|X_1^{d*}\|^2 < \|X_0^d\|^2) = 2\mathbb{P}(\|X_0^d + \sigma_d W_1^d\|^2 < \|X_0^d\|^2) \\ &= 2\mathbb{P}(\langle X_0^d, W_1^d\rangle < -\sigma_d \|W_1^d\|^2/2).\end{aligned}$$

In many cases, $\langle X_0^d, W_1^d\rangle = O(d^{1/2})$ and $\|W_1^d\|^2 = O(d)$. Thus, we need to take $\sigma_d = O(d^{-1/2})$ to avoid degeneracy, which results in at least d iteration for convergence, and so d^2 computational cost. Many technologies developed to suppress or weaken the effect of dimensionality. High-dimensional asymptotics give some insight in this direction.

We review some recent theoretical developments in high-dimensional asymptotics for Markov chain Monte Carlo methods. We also discuss a new algorithm [2] that is considered to be a quasi Hamiltonian Monte Carlo method. Finally, we discuss an application to the R-package, yuima.

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References

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