On estimation of intrinsic stationary random fields

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We propose estimators of isotropic intrinsic stationary random fields by log-periodogram regression and derive their asymptotic properties. These are extensions of the results for stationary and nonstationary long-memory time series models given by Robinson(1995.*Ann.Statist.*) and Velasco(1999.*J.Econometrics*) to random fields.

Let $\{X(\boldsymbol{t});\boldsymbol{t}\in \boldsymbol{R}^d\}$ be a random field. If for any fixed $\boldsymbol{h}(\in \boldsymbol{R}^d)$, the random field $\{Z_{\boldsymbol{h}}(\boldsymbol{t});\boldsymbol{t}\in \boldsymbol{R}^d\}$ defined by

$$Z_{\boldsymbol{h}}(\boldsymbol{t}) = X(\boldsymbol{t}+\boldsymbol{h}) - X(\boldsymbol{t}),$$

is stationary, $\{X(\boldsymbol{t});\boldsymbol{t}\in \boldsymbol{R}^d\}$ is called an intrinsic stationary random field. Then the variogram $\gamma(\boldsymbol{h})$ is defined by

$$\gamma(\boldsymbol{h}) = \mathrm{Var}(X(\boldsymbol{t}+\boldsymbol{h}) - X(\boldsymbol{t}))$$

and under some assumptions, it has the spectral representation,

$$\gamma(\boldsymbol{h}) = \int_{\boldsymbol{R}^d} \frac{1-\cos((\boldsymbol{\omega},\boldsymbol{h}))}{(2\pi)^d} g(\boldsymbol{\omega})d\boldsymbol{\omega}$$

where $g(\boldsymbol{\omega})$ is positive and satisfies

$$\int_{\boldsymbol{R}^d} \frac{\| \boldsymbol{\omega} \|^2}{1+\| \boldsymbol{\omega} \|^2} g(\boldsymbol{\omega})d\boldsymbol{\omega} < \infty.$$

Hereafter for simplicity we assume $d = 2$ and the sampling sites are

$$s_{qr} = (q,r),\ \ q,r = 1,\ldots,n.$$

The tapered discrete Fourier transform and the periodogram are defined by

$$w_p^T(\omega_{j_1j_2}) = \frac{1}{2\pi\sum_{t=1}^n h_t^2}\sum_{q,r=1}^{n} h_q h_r X(s_{qr})\exp(-i(\omega_{j_1j_2}, s_{qr}))$$
$$I_p^T(\omega_{j_1j_2}) = |w_p^T(\omega_{j_1j_2})|^2$$

where

$$\omega_{j_1j_2} = (\omega_{j_1},\omega_{j_2}) = (\frac{2\pi j_1}{n},\frac{2\pi j_2}{n}),\ -\frac{n}{2} < j_1,\ j_2 \le \frac{n}{2}$$

and h_t is a positive taper symmetric around $n/2$ with $\max_t h_t = 1$ and of order p. We assume that $g(\lambda_1,\lambda_2)$ depends only on $\lambda_1^2+\lambda_2^2$, that is, isotropic and $g(\lambda_1,\lambda_2)/(\lambda_1^2+\lambda_2^2)^{-(H+1)}$ with $0 < H < 1$ is bounded and bounded away from 0. We apply a log-periodogram regression to $I_p^T(\omega_{j_1j_2})$ to estimate H. Then we derive its asymptotic properties and conduct some computational experiments that compare the performance of this estimator and that of estimators proposed previously by other authors.