Weak Admissibility in High-dimensional and Nonparametric Statistical Models

Keisuke Yano, The University of Tokyo
Fumiyasu Komaki, The University of Tokyo, RIKEN Brain Science Institute

1. Introduction

This talk focuses on the use of ε-admissibility (weak admissibility) in high-dimensional and nonparametric statistical models. The minimax rate of convergence is widely used as a criterion of comparing performance of estimators in high-dimensional and nonparametric statistical models. However, there often appear settings in which the minimax rate of convergence is no use to compare the performance of estimators or does not reflect on the knowledge in the finite-dimensional models. In such settings, an additional comparison by ε-admissibility is helpful. We demonstrate the usefulness of ε-admissibility through several examples by presenting new results.

2. Definition of weak admissibility

Consider a statistical decision problem: Let $\mathcal{X}$ be a sample space; Let Θ be a parameter space; Let $\mathcal{P}$ be a statistical model $\{P_\theta : \theta \in \Theta\}$; Let $\mathcal{A}$ be an action space; Let $\mathcal{D}$ be a decision space, the whole set of functions from $\mathcal{X}$ to $\mathcal{A}$; Let L be a loss function $\Theta\times \to \mathbb{R} \cup \{+\infty\}$; Let R be the risk function $R(\theta, \delta) = \int L(\theta, \delta(X))\mathrm{d}P_\theta(X)$.

For the statistical decision problem, an estimator δ is ε-admissible if and only if there does not exist an estimator $\tilde{\delta}$ such that

$$R(\theta, \tilde{\delta}) < R(\theta, \delta) - \varepsilon \text{ for all } \theta \in \Theta.$$

For the details, see [1, 2].

References

[1] Ferguson, T. (1967). *Mathematical Statistics: A Decision Theoretic Approach.* Academic Press.

[2] Hartigan, J. (1983). *Bayes theory.* Springer