

シリンダー上の Pareto 型分布

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1. Introduction

A probability distribution on the cylinder or cylindrical distribution refers to a bivariate distribution with a linear random variable X and a circular (angular) random variable Θ . Examples of cylindrical observations include wind speed and wind direction, wave height and wave direction in environmental and marine sciences, birth rate and time in the area of biological rhythms, and distance and direction in the areas of animal movement and seismology. Johnson and Wehrly (1978) have proposed a cylindrical distribution whose marginal distribution of Θ is a wrapped Cauchy distribution, conditional distribution of X given $\Theta = \theta$ is an exponential distribution, and conditional distribution of Θ given $X = x$ is a von Mises distribution. The marginal distribution of X “does not follow a familiar distribution (Johnson and Wehrly, 1978).” We propose an extension of this distribution.

2. Model

We propose a Pareto-type distribution on the cylinder with the following joint probability density function:

$$f(x, \theta) = \frac{\gamma\sqrt{1-\kappa^2}}{2\pi\sigma} \left(\frac{x}{\sigma}\right)^{\gamma-1} \left[1 + \xi \left(\frac{x}{\sigma}\right)^\gamma \{1 - \kappa \cos(\theta - \mu)\}\right]^{-(1/\xi+1)}$$

for $\gamma > 0$, $\sigma > 0$, $0 \leq \mu < 2\pi$, $0 \leq \kappa < 1$ and $\xi \neq 0$, where $x > 0$ and $0 \leq \theta < 2\pi$ when $\xi > 0$, and $0 < x < \sigma/[-\xi\{1 - \kappa \cos(\theta - \mu)\}]^{1/\gamma}$ and $0 \leq \theta < 2\pi$ when $\xi < 0$. The parameter γ is a linear shape and σ a linear scale. It is easy to see that μ is a circular location parameter, and the distribution is symmetric about $\theta = \mu$ for fixed x . The parameter κ is a circular concentration. If $\kappa = 0$ as a special case, then X is independent of Θ , and X follows a generalized Pareto distribution and Θ a circular uniform distribution. The parameter ξ controls tail behavior and shape. The distribution extends the symmetric case of the WeiSSVM distribution by Abe and Ley (2017).

For the proposed distribution with $\gamma = 1$, we observe the following. For $\xi = 0$ by continuity, we have the distribution by Johnson and Wehrly (1978) as a special case. The marginal distribution of Θ is a wrapped Cauchy distribution, the conditional distribution of X given $\Theta = \theta$ is a generalized Pareto distribution, and the conditional distribution of Θ given $X = x$ belongs to the t -distribution on the circle (Shimizu and Iida, 2002). The marginal distribution of X can be interpreted as a mixture of beta distribution of the second kind and negative binomial.

References

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