Lévy-driven CARMA Random Fields on R^n

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Abstract: We define an isotropic Lévy-driven CARMA(p, q) random field on R^n as the integral of an isotropic CARMA kernel with respect to a Lévy sheet. Such fields constitute a parametric family characterized by an autoregressive polynomial a and a moving average polynomial b having zeros in both the left and right complex half-planes. They extend the *well-balanced Ornstein-Uhlenbeck process* of Schnurr and Woerner (2011) to a well-balanced CARMA process in one dimension (with a much richer class of autocovariance functions) and to an isotropic CARMA random field on R^n for $n > 1$. We derive second-order properties of these random fields and find that CAR(1) constitutes a subclass of the well known Matérn class. If the driving Lévy sheet is compound Poisson it is a trivial matter to simulate the corresponding random field on any n-dimensional hypercube. Joint estimation of CARMA kernel parameters and knots locations is proposed in cases driven by compound Poisson sheets and is illustrated by applications to land price data in Tokyo as well as simulated data.

Key words and phrases: compound Poisson, convolution, CARMA random field, Gibbs sampling, knot selection, Lévy sheet, Lévy noise, Matérn class.