On constructing a hypothesis test for multivariate Gaussian random function

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Gaussian random function is a random function whose values are normally distributed and has some good properties (e.g., Lifshits [2], Tarpey and Kinateder [5], Shimizu and Mizuta [4]). In this presentation, we discuss hypothesis testing for multivariate Gaussian random function.

A random function can be viewed as an infinite ($p = \infty$) dimensional random variable even if it is a univariate random function. Recently, Himeno and Yamada [1] constructed a normality test for high-dimensional distributions that tests whether a p-dimensional distribution F is normal or not, using p-dimensional vectors $\boldsymbol{X}_1, \ldots, \boldsymbol{X}_n$ that are a random sample of size n. Their test is applicable even if $n < p$.

Based on the results of Himeno and Yamada [1], Matsuura, Yamashita, and Kinateder [3] proposed a test for whether a univariate random function is a univariate Gaussian random function or not.

In this presentation, we extend it to a test for multivariate Gaussian random function. We propose a procedure to test whether a p-dimensional random function $\boldsymbol{X}(t),\ 0 < t < T$ is a multivariate Gaussian random function or not, using p-dimensional functional data $\boldsymbol{X}_1(t), \ldots, \boldsymbol{X}_n(t),\ 0 < t < T$ that are a random sample of size n. We show differences between the test of Himeno and Yamada [1] and our test. We also provide some numerical results for the size and the power of our test.

References

[1] Himeno, T., Yamada, T. (2014), "Estimations for some functions of covariance matrix in high dimension under non-normality and its applications", *Journal of Multivariate Analysis*, Vol. 130, pp. 27-44.

[2] Lifshits, M. A. (1995), *Gaussian Random Functions*, Springer.

[3] Matsuura, S., Yamashita, H., Kinateder, K. K. J., "On statistical hypothesis testing for Gaussian random function", Kyoto International Conference on Modern Statistics in the 21st Century, Kyoto, Japan, November 2014.

[4] Shimizu, N., Mizuta, M. (2008), "Functional principal points and functional cluster analysis", In: Jain, L. C., Sato-Ilic, M., Virvou, M., Tsihrintzis, G. A., Balas, V. E., Abeynayake, C. (Eds.), *Computational Intelligence Paradigms: Innovative Applications*, Studies in Computational Intelligence, Vol. 137, pp. 149-165.

[5] Tarpey, T., Kinateder, K. (2003), "Clustering functional data", *Journal of Classification*, Vol. 20, No. 1, pp. 93-114.