An extension of factor rotation via the penalized maximum likelihood estimation

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In exploratory factor analysis, a traditional estimation procedure in use is the following two-step approach: a model fitting is done by the maximum likelihood estimation, and then rotation techniques, such as the varimax method or the promax method, are utilized to obtain an interpretable loading matrix. In practical situations, however, the two-step approach as mentioned above does not often perform well for small sample sizes because of the overparameterization. In such cases, a penalized maximum likelihood procedure that produces sparse solutions, such as the lasso, can handle these issues (Hirose and Yamamoto, in press). Although many rotation criteria can be used as penalties in the penalized maximum likelihood estimation, Hirose and Yamamoto only applied the lasso-type penalties such as the minimax concave penalty (Zhang, 2010).

In practice, the lasso does not often perform well in terms of the interpretation because estimating some coefficients *exactly* zero can often produce inappropriate estimates of nonzero loadings. On the other hand, the traditional rotation criteria, such as the varimax and the quartimax, can make some of the factor loadings nearly (not exactly) zero. These traditional rotation criteria can often produce an interpretable model. However, the varimax-type criterion must be *maximized* under some constraint. If the varimax-type criterion is used as a penalty of the penalized maximum likelihood procedure, the estimates of factor loadings become larger than the maximum likelihood estimates. Estimating coefficients that are too large is opposed to the basic concept of the penalized maximum likelihood procedure; the penalized maximum likelihood procedure usually shrinks some coefficients toward zero to produce stable estimates.

In the present paper, we introduce a new penalty that shares the strength of both the lasso and the varimax-type criterion. We use a rotation criterion that is minimized when the loading matrix is a zero matrix under no constraint, such as the quartimin and the covarimin criteria. The proposed penalty shrinks some coefficients toward zero and also produces an interpretable loading matrix. The details of the proposed penalty term, estimation algorithm, and the numerical results are given in the presentation.

References

Hirose, K. and Yamamoto, M. Sparse estimation via nonconcave penalized likelihood in a factor analysis model. *Statistics and Computing*, in press.

Zhang, C. H. (2010). Nearly unbiased variable selection under minimax concave penalty. *The Annals of Statistics*, 894–942.