Cholesky Realized Stochastic Volatility Model

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Multivariate stochastic volatility models are expected to play important roles in financial applications such as asset allocation and risk management. However, these models suffer from two major difficulties: (1) there are too many parameters to estimate using only daily asset returns and (2) estimated covariance matrices are not guaranteed to be positive definite. Our approach takes advantage of realized covariances to attain the efficient estimation of parameters by incorporating additional information for the co-volatilities, and considers Cholesky decomposition to guarantee the positive definiteness of the covariance matrices:

$$\boldsymbol{y}_t \sim \mathcal{N}(\boldsymbol{m}_t, \boldsymbol{\Sigma}_t), \quad t = 1, \ldots, n \tag{1}$$

$$\boldsymbol{\Sigma}_t = \mathbf{H}_t^{*-1}\mathbf{V}_t\mathbf{H}_t^{*-1'}, \quad \mathbf{V}_t = \mathrm{diag}(\exp(h_{11,t}), \ldots, \exp(h_{pp,t})), \tag{2}$$

$$\mathbf{H}_t^* = \begin{pmatrix} 1 & 0 & 0 & \ldots & 0 \\ -h_{21,t} & 1 & 0 & \ldots & 0 \\ -h_{31,t} & -h_{32,t} & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -h_{p1,t} & -h_{p2,t} & \ldots & -h_{p\,p-1,t} & 1 \end{pmatrix}. \tag{3}$$

where $\boldsymbol{y}_t = (y_{1t}, \ldots, y_{pt})'$ denotes a $p \times 1$ assets return vector and $\boldsymbol{\Sigma}_t = \mathbf{H}_t^{*-1}\mathbf{V}_t\mathbf{H}_t^{*-1'}$ is Cholesky decomposition of the covariance matrix of $\boldsymbol{y}_t$ given h_{ijt}'s.

In this framework, we propose a flexible modeling for stylized facts of financial markets such as dynamic correlations and leverage effects among volatilities. Taking a Bayesian approach, we describe Markov Chain Monte Carlo implementation with a simple but efficient sampling scheme. Our model is applied to nine U.S. stock returns data, and the model comparison is conducted based on portfolio performances. In our empirical studies, our proposed CRSV models capture the dynamic behaviors of diagonal and off-diagonal components of Cholesky decomposed covariance matrices and are shown to outperform CSV models.