Testing for Granger Causality with Mixed Frequency Data
(Short Version)†

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Abstract

It is well known that temporal aggregation has adverse effects on Granger causality tests. Time series are often sampled at different frequencies, but this is typically ignored and data are merely aggregated to the common lowest frequency. We develop a set of Granger causality tests that explicitly take advantage of data sampled at different frequencies. We show that taking advantage of mixed frequency data allows us to better recover causal relationships when compared to the conventional common low frequency approach. We also show that the mixed frequency causality tests have higher local asymptotic power as well as more power in finite samples compared to conventional tests. In an empirical application involving U.S. macroeconomic indicators, we show that the mixed frequency approach and the low frequency approach produce very different causal implications, with the former yielding more intuitive result.

Keywords: Granger causality test, Local asymptotic power, Mixed data sampling (MIDAS), Temporal aggregation, Vector autoregression (VAR)

1 Introduction

It is well known that temporal aggregation may have spurious effects on testing for Granger’s (1969) causality, as noted by C. Granger himself in many papers, see e.g. Granger (1980). In this paper we deal with what might be an obvious, yet largely overlooked remedy. Time series are often sampled at different frequencies and then typically aggregated to the common lowest frequency to test for causality. The analysis of the present paper pertains to comparing testing for causality with all series aggregated to the common lowest frequency, and testing for causality taking advantage of all the series sampled at whatever frequency they are available. We use mixed frequency vector autoregressive models to implement a new class of Granger causality tests.1

We show that mixed frequency Granger causality tests better recover causal patterns in an underlying high frequency process compared to the traditional low frequency approach. We also show that mixed frequency

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1MIDAS, meaning Mi(xed) Da(ta) S(ampling), regression models have been put forward in recent work by Ghysels, Santa-Clara, and Valkanov (2004), Ghysels, Santa-Clara, and Valkanov (2006), and Andreou, Ghysels, and Kourtellos (2010). VAR models for mixed frequency data were independently introduced by Anderson, Deistler, Felsenstein, Funovits, Zadrozny, Eichler, Chen, and Zamani (2012), Ghysels (2012), and McCracken, Owyang, and Sekhposyan (2013).
causality tests have higher asymptotic power against local alternatives and show via simulation that this also holds in finite samples involving realistic data generating processes.

We apply the mixed frequency causality test to monthly inflation, monthly crude oil price fluctuations, the real GDP growth in the U.S. We also apply the conventional low frequency causality test to the aggregated quarterly price series and real GDP for comparison. These two approaches yield very different causal implications, and the mixed frequency approach yields more intuitive results.

The paper is organized as follows. In Section 2 we frame mixed frequency VAR models. In Section 3 we develop the mixed frequency Granger causality tests. Section 4 discusses recovery of underlying causality. Section 5 conducts local asymptotic power analysis. Section 6 reports Monte Carlo simulation results. Section 7 presents empirical application. Section 8 concludes the paper.

2 Mixed Frequency Data Model Specifications

This section frames a mixed frequency vector autoregressive (MF-VAR) model and present some asymptotic properties. We want to characterize three settings which we will refer to as HF, MF and LF - respectively high, mixed and low frequency. We begin by considering a partially latent underlying HF process. Using the notation of Ghysels (2012), the HF process contains \( \{x_H(\tau_L, k)\}_{k=1}^{m} \) and \( \{x_L(\tau_L, k)\}_{k=1}^{m} \), where \( \tau_L \in \{0, \ldots, T_L\} \) is the LF time index (e.g. quarter), \( k \in \{1, \ldots, m\} \) denotes the HF (e.g. month), and \( m \) is the number of HF time periods within each LF time period. In the month vs. quarter case, for example, \( m \) equals three as one quarter has three months. This short paper assumes that there are only one high frequency variable \( x_H \) and only one low frequency variable \( x_L \). The full paper treats an arbitrary number of variables.

Note that \( x_H(\tau_L, k) \) is observable but \( x_L(\tau_L, k) \) is not in reality - only aggregated \( x_L \) is observable. The analyst’s practical choice, in other words, is limited to MF and LF cases. Only low frequency variable \( x_L \) is aggregated from a latent HF process in a MF setting, whereas both \( x_L \) and \( x_H \) are aggregated from the latent HF process to form a LF process. We consider linear aggregation schemes involving weights \( w = [w_1, \ldots, w_m]' \) such that \( x_H(\tau_L) = \sum_{k=1}^{m} w_k x_H(\tau_L, k) \) and \( x_L(\tau_L) = \sum_{k=1}^{m} w_k x_L(\tau_L, k) \). Two cases are of special interest given their broad use: (1) stock or skipped sampling, where \( w_k = I(k = m) \); and (2) flow sampling, where \( w_k = 1/m \) for \( k = 1, \ldots, m \). In summary, we observe:

- both high and low frequency variables \( \{x_H(\tau_L, j)\}_{j=1}^{m} \) and \( \{x_L(\tau_L, j)\}_{j=1}^{m} \) in a HF process;
- high frequency variable \( \{x_H(\tau_L, j)\}_{j=1}^{m} \) but only aggregated low frequency variable \( x_L(\tau_L) \) in a MF process;
- only aggregated high and low frequency variables \( x_H(\tau_L) \) and \( x_L(\tau_L) \) in a LF process.

A key idea of MF-VAR models is to stack everything observable given a MF process according to their order over time. This results in the \( K = m + 1 \) dimensional mixed frequency vector \( X(\tau_L) = [x_H(\tau_L, 1), x_H(\tau_L, 2), \ldots, x_H(\tau_L, m), x_L(\tau_L)]' \). Note that \( x_L(\tau_L) \) is the last block in the stacked vector - a conventional assumption that \( x_L(\tau_L) \) is observed after \( x_H(\tau_L, m) \). Any other order is conceptually the same, except that it implies a different timing of information about the respective processes. We will work with the specification appearing here as it is most convenient.

We will make a number of standard regulatory assumptions. Let \( F_{\tau_L} = \sigma(X(\tau): \tau \leq \tau_L) \). In particular we assume \( E[X(\tau_L)|F_{\tau_L-1}] \) has a version that is almost surely linear in \( \{X(\tau_L-1), \ldots, X(\tau_L-p)\} \) for some finite \( p \geq 1 \): \( X(\tau_L) = \sum_{k=1}^{p} A_k X(\tau_L- k) + \epsilon(\tau_L) \). This is a fundamental assumption of Ghysels’ (2012) MF-VAR model; it enables us to work on mixed frequency data without dealing with latent variables. The
coefficient\(A_k\) are \(K \times K\) matrices for \(k = 1, \ldots, p\). The error vector \(\epsilon(\tau_L)\) is a strictly stationary martingale difference with respect to increasing \(F_{\tau_L} \subset F_{\tau_L+1}\), where \(\Omega = E[\epsilon(\tau_L)\epsilon(\tau_L)']\) is positive definite.

In addition, we assume that all roots of the polynomial \(\det(I_k - \sum_{k=1}^p A_k z^k) = 0\) lie outside the unit circle. This condition ensures the stability of MF-VAR process. Also, we assume that \(X(\tau_L)\) and \(\epsilon(\tau_L)\) are \(\alpha\)-mixing (see the full paper for more details).

Given these assumptions, it is straightforward to prove consistency and asymptotic normality of the ordinary least squares estimator of \(A_1, \ldots, A_p\). Linear parametric restrictions of \(A_1, \ldots, A_p\) can be tested by Wald tests as usual. The asymptotic distribution of the Wald statistic under the null hypothesis is \(\chi^2_q\), where \(q\) is the number of restrictions. See the full paper for more formal asymptotic theory.

### 3 Testing Causality with Mixed Frequency Data

In this section we define non-causality when data are sampled at mixed frequencies and describe Wald tests associated with it. This short paper works on a bivariate setting where we have only one high frequency variable \(x_H\) and only one low frequency variable \(x_L\) (the full paper handles an arbitrary number of variables).

Recall the \(K \times 1\) mixed frequency vector \(X(\tau_L) = [x_H(\tau_L, 1), \ldots, x_H(\tau_L, m), x_L(\tau_L)]'\). We write \(\tilde{x}_H(\tau_L) = [x_H(\tau_L, 1), \ldots, x_H(\tau_L, m)]'\) so that we have \(X(\tau_L) = (\tilde{x}_H(\tau_L), x_L(\tau_L))'\). Let \(X(-\infty, \tau_L)\) be the Hilbert space spanned by \(\{X(\tau)\}_{\tau \leq \tau_L}\). \(\tilde{x}_H(-\infty, \tau_L)\) and \(x_L(-\infty, \tau_L)\) are defined analogously. We call \(\mathcal{I} = \{X(-\infty, \tau_L) | \tau_L \in \mathbb{Z}\}\) the mixed frequency information set. Using these information sets, we can define Granger non-causality between \(x_L\) and \(x_H\).

**Definition 3.1.** High frequency variable \(x_H\) does not Granger cause low frequency variable \(x_L\) given the mixed frequency information set \(\mathcal{I} = \{X(-\infty, \tau_L) | \tau_L \in \mathbb{Z}\}\) (denoted by \(x_H \rightarrow x_L | \mathcal{I}\)) if \(P[x_L(\tau_L+1) | x_L(-\infty, \tau_L)] = P[x_L(\tau_L+1) | X(-\infty, \tau_L)]\), where \(P[x(\tau+1) | I(\tau)]\) is the best linear projection of \(x(\tau+1)\) conditional on \(I(\tau)\). Similarly, \(x_L\) does not Granger cause \(x_H\) given \(\mathcal{I}\) (denoted by \(x_L \rightarrow x_H | \mathcal{I}\)) if \(P[\tilde{x}_H(\tau_L+1) | \tilde{x}_H(-\infty, \tau_L)] = P[\tilde{x}_H(\tau_L+1) | X(-\infty, \tau_L)]\).

As is well-known in Dufour and Renault (1998) among others, non-causality in Definition 3.1 can be expressed as zero restrictions in VAR. Recall that our model is \(X(\tau_L) = \sum_{k=1}^p A_k X(\tau_L - k) + \epsilon(\tau_L)\) with

\[
A_k = \begin{bmatrix}
a_{11,k} & \cdots & a_{1m,k} & a_{1K,k} \\
\vdots & \ddots & \vdots & \vdots \\
a_{m1,k} & \cdots & a_{mm,k} & a_{mK,k} \\
a_{K1,k} & \cdots & a_{KK,m} & a_{KK,k}
\end{bmatrix} = \begin{bmatrix}A_{HH,k} & a_{HL,k} \\a_{LH,k} & a_{KK,k}\end{bmatrix}.
\]

It follows that \(x_H \rightarrow x_L | \mathcal{I}\) if and only if \(a_{LH,k} = 0_{m \times 1}\) for all \(k = 1, \ldots, p\). Similarly, \(x_L \rightarrow x_H | \mathcal{I}\) if and only if \(a_{HH,k} = 0_{m \times 1}\) for all \(k = 1, \ldots, p\). These zero restrictions can be tested easily by linear Wald tests, and the asymptotic distribution under non-causality is \(\chi^2_{pm}\) as discussed in the end of Section 2.

### 4 Recovery of High Frequency Causality

The existing literature on Granger causality and temporal aggregation has three key ingredients. Starting with (1) a data generating process (DGP) for HF data, and (2) specifying a (linear) aggregation scheme, one is interested in (3) the relationship between causal patterns - or lack thereof - among the HF series and the inference obtained from LF data when all HF series are aggregated. So far, we refrained from (1) specifying
a DGP for HF series and (2) specifying an aggregation scheme. We will proceed along the same path as the existing literature in this section with a different purpose, namely to show that the MF approach recovers more underlying causal patterns than the standard LF approach does. While conducting Granger causality tests with MF series does not reveal all HF causal patterns, using MF instead of using exclusively LF series promotes sharper inference.

In this short paper we consider bivariate HF-VAR(1) with stock sampling, in which case we can analytically derive the implied MF-VAR(1) and LF-VAR(1). More general discussion involving an arbitrary HF-VAR lag order and linear aggregation scheme can be found in the full paper.

Let \( \mathbf{X}(\tau_L, k) = [x_H(\tau_L, k), x_L(\tau_L, k)]' \), and we assume that this vector follows high frequency VAR(1):

\[
\mathbf{X}(\tau_L, k) = \Phi_1 \mathbf{X}(\tau_L, k - 1) + \eta(\tau_L, k), \quad \text{where} \quad \Phi_1 = \begin{bmatrix} \phi_{HH,1} & \phi_{HL,1} \\ \phi_{LH,1} & \phi_{LL,1} \end{bmatrix}.
\] (4.1)

It is understood that \( \mathbf{X}(\tau_L, 0) = \mathbf{X}(\tau_L - 1, m). \) \( \eta(\tau_L, k) \) is a stationary martingale difference. Assuming stock sampling, we simply have that \( x_L(\tau_L) = x_L(\tau_L, m) \). This implies that a MF-VAR process corresponding to (4.1) is \( \mathbf{X}(\tau_L) = \mathbf{A}_1 \mathbf{X}(\tau_L - 1) + \epsilon(\tau_L) \) with

\[
\mathbf{A}_1 = \begin{bmatrix} 0_{1 \times (m-1)} & \cdots & \phi_{\tau_H,1}^{[1]} & \phi_{\tau_H,1}^{[2]} \\ \vdots & \ddots & \vdots & \cdots \\ 0_{1 \times (m-1)} & \cdots & \phi_{\tau_H,1}^{[m]} & \phi_{\tau_H,1}^{[m]} \\ 0_{1 \times (m-1)} & \cdots & \phi_{\tau_L,1}^{[m]} & \phi_{\tau_L,1}^{[m]} \end{bmatrix}, \quad \text{where} \quad \Phi_1^{[m]} = \begin{bmatrix} \phi_{H1,1}^{[1]} & \phi_{H1,1}^{[2]} \\ \phi_{L1,1}^{[1]} & \phi_{L1,1}^{[2]} \end{bmatrix}.
\] (4.2)

Similarly, we define \( \mathbf{X}(\tau_L) = [x_H(\tau_L), x_L(\tau_L)]' \) with stock sampling \( x_H(\tau_L) = x_H(\tau_L, m). \) Then a LF-VAR process corresponding to (4.1) is \( \mathbf{X}(\tau_L) = \mathbf{A}_1 \mathbf{X}(\tau_L - 1) + \epsilon(\tau_L) \) with

\[
\mathbf{A}_1 = \Phi_1^{[m]}.
\] (4.3)

It can be shown that error terms \( \epsilon(\tau_L) \) and \( \epsilon(\tau_L) \) are stationary martingale difference sequences. See the full paper for more detailed derivations of (4.2) and (4.3).

We have already defined non-causality given mixed frequency information set \( \mathbb{I} \) as the Hilbert space spanned by \( \{\mathbf{X}(\tau_L)\} \). Analogously we define high frequency information set \( \hat{\mathbb{I}} \) and low frequency information set \( \tilde{\mathbb{I}}. \) \( \hat{\mathbb{I}} \) is the Hilbert space spanned by high frequency vector \( \{\mathbf{X}(\tau_L, k)\} \), while \( \tilde{\mathbb{I}} \) is the Hilbert space spanned by low frequency vector \( \{\mathbf{X}(\tau_L)\} \). Equations (4.2) and (4.3) tell us how Granger causality given each information set is transferred after stock aggregation. First, non-causality from \( x_L \) to \( x_H \) given \( \mathbb{I} \) (i.e. \( \phi_{H1,1} = 0 \)) implies non-causality given \( \hat{\mathbb{I}} \) (i.e. \( \phi_{H1}^{[1]} = 0 \)) as well as non-causality given \( \tilde{\mathbb{I}} \). Second, non-causality from \( x_L \) to \( x_H \) given \( \mathbb{I} \) does not imply non-causality given \( \hat{\mathbb{I}} \). Since having \( \phi_{H1}^{[m]} = 0 \) does not imply \( \phi_{H1,1} = 0 \), non-causality from \( x_L \) to \( x_H \) given \( \mathbb{I} \) implies non-causality given \( \tilde{\mathbb{I}} \). Repeating similar arguments for high-to-low causality, the following implications hold.

- Consider Granger causality from \( x_H \) to \( x_L \). Non-causality given high frequency information set \( \hat{\mathbb{I}} \) implies non-causality given mixed frequency information set \( \mathbb{I} \), which is necessary and sufficient for non-causality given low frequency information set \( \tilde{\mathbb{I}} \).

- Consider Granger causality from \( x_L \) to \( x_H \). Non-causality given \( \tilde{\mathbb{I}} \) is necessary and sufficient for non-causality given \( \mathbb{I} \), which implies non-causality given \( \hat{\mathbb{I}} \).

The first item shows that no relevant information for testing high-to-low causality is lost when moving
from $\mathcal{I}$ to $\mathcal{L}$, while some information is lost when moving from $\mathcal{I}$ to $\mathcal{I}$. The second item shows that no relevant information for testing low-to-high causality is lost when moving from $\mathcal{I}$ to $\mathcal{L}$, while some information is lost when moving from $\mathcal{I}$ to $\mathcal{I}$. These two results suggest that the mixed frequency causality test should never perform worse than the low frequency causality test (up to parameter proliferation), and the former should be more powerful than the latter when low-to-high causality is of interest. Sections 5 and 6 verify this point by local asymptotic power analysis and Monte Carlo simulations, respectively.

5 Local Asymptotic Power Analysis

This section compares the local asymptotic power of the mixed frequency causality test and low frequency causality test. We need to constrain our attention to analytically tractable DGPs, which is why we consider a bivariate HF-VAR(1) process with stock sampling. As shown in Section 4, for the bivariate HF-VAR(1) one can derive the corresponding MF- and LF-VAR(1) processes explicitly. We focus on Granger causality from $x_L$ to $x_H$ below. The opposite direction can be handled analogously.

Assume that the high frequency DGP is given by (4.1) with

$$\Phi_1 = \Phi(\nu/\sqrt{T}) = \begin{bmatrix} \rho_H & \nu/\sqrt{T} \\ 0 & \rho_L \end{bmatrix}.$$  

We assume that $\rho_H, \rho_L \in (-1, 1)$. $\nu \in \mathbb{R}$ is the usual Pitman drift parameter. Sample size $T = m \times T_L$ is in terms of high frequency. Assume for computational simplicity that $(\tau_L, k)$ $\overset{m.d.s.}{\sim} (0_{2 \times 1}, I_2)$. In the true DGP, the high frequency variable does not cause the low frequency variable, while for $\nu \neq 0$ the low frequency variable causes the high frequency variable with a marginal impact of $\nu/\sqrt{T}$ which vanishes as $T \to \infty$.

Assuming stock sampling and general $m \in \mathbb{N}$, (4.2) implies that the corresponding MF-VAR(1) process is characterized by

$$A_1 = A(\nu/\sqrt{T}) = \begin{bmatrix} 0_{1 \times (m-1)} & \rho_H & \sum_{k=1}^1 \rho_H^{k-1}\rho_L^{1-k}(\nu/\sqrt{T}) \\ \vdots & \vdots & \vdots \\ 0_{1 \times (m-1)} & \rho_H^m & \sum_{k=1}^m \rho_H^{k-1}\rho_L^{m-k}(\nu/\sqrt{T}) \\ 0_{1 \times (m-1)} & 0 & \rho_L^m \end{bmatrix}. $$

We fit a MF-VAR(1) model, which is correctly specified, and then implement the MF low-to-high causality test. The limit distribution of the Wald statistic under the null hypothesis of non-causality is $\chi^2_m$, while the limit distribution under the local alternative hypothesis is $\chi^2_m(\kappa_{MF})$, the noncentral chi-squared distribution with degrees of freedom $m$ and noncentrality parameter $\kappa_{MF}$ (see the full paper for an analytical expression of $\kappa_{MF}$). The local asymptotic power can be computed through the cumulative distribution functions of these limit distributions.

The local asymptotic power of the low frequency low-to-high causality test can be derived analogously. First, (4.3) implies that the LF-VAR(1) process corresponding to (4.1) is characterized by

$$A_1 = A(\nu/\sqrt{T}) = \begin{bmatrix} \rho_H^m & \sum_{k=1}^m \rho_H^{k-1}\rho_L^{m-k}(\nu/\sqrt{T}) \\ 0 & \rho_L^m \end{bmatrix}. $$

We fit a LF-VAR(1) model and implement the classic low frequency low-to-high causality test. The local power can be calculated through the c.d.f. of the asymptotic distribution under the null (i.e. $\chi^2_1$) and the local alternative (i.e. $\chi^2_1(\kappa_{LF})$). See the full paper for an explicit formula of $\kappa_{LF}$.
The local asymptotic power associated with high-to-low causality can be calculated analogously. The only difference is that we start with HF-VAR(1) having a coefficient matrix $\Phi(\nu/\sqrt{T})'$, not $\Phi(\nu/\sqrt{T})$.

**Numerical Exercises**  Figure 1 presents local asymptotic power of the mixed frequency causality test and low frequency causality test in a concrete setting. Panel A considers Granger causality from $x_H$ to $x_L$, while Panel B considers Granger causality from $x_L$ to $x_H$. Each panel plots local asymptotic power associated with (a) mixed frequency test and (b) conventional low frequency test. Ratio of sampling frequencies, $m$, is taken between three (e.g. month vs. quarter) and twelve (e.g. month vs. year). Pitman parameter $\nu$ is taken between 3.5 and 4.5 for high-to-low causality, while it is taken between 0.5 and 1.5 for low-to-high causality. We assume $(\rho_H, \rho_L) = (0.25, 0.75)$, i.e. low persistence in $x_H$ and high persistence in $x_L$. See the full paper for other parametrizations.

Panel A suggests that the mixed frequency test is roughly as powerful as the low frequency test when causality from $x_H$ to $x_L$ is concerned. The former is in fact slightly less powerful than the latter due to the larger number of parameters. Both tests achieve power about 60% when $m$ is small, but they lose power as $m$ increases.

Panel B highlights an advantage of the mixed frequency test over the low frequency test. Fixing the Pitman parameter $\nu = 1.5$, the mixed frequency test achieves power about 60-70% regardless of $m$. The low frequency
test, in contrast, achieves power about 40% for small $m$ and lower and lower power as $m$ increases. When $m = 12$, the low frequency test has absolutely no power while the mixed frequency test still has power about 60%. This result suggests that taking advantage of mixed frequency data improves the accuracy of causality test dramatically, when causality from $x_L$ to $x_H$ is concerned.

6 Power Improvements in Finite Samples

This section conducts Monte Carlo simulations for bivariate cases to evaluate the finite sample performance of the mixed frequency causality test relative to the low frequency test. In particular, this section considers a bivariate HF-VAR(1) process with stock sampling so that the corresponding MF- and LF-VAR processes are known. Trivariate cases are covered in the full paper but omitted here.

Simulation Design

We draw 1,000 independent samples from a HF-VAR(1) process $\{\mathbf{X}(\tau_L, k)\}$ with coefficient $\Phi_1$ partitioned in two possible ways:

(a) $\begin{bmatrix} \phi_{HH,1} & \phi_{HL,1} \\ \phi_{LH,1} & \phi_{LL,1} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.0 \\ 0.35 & 0.4 \end{bmatrix}$ and (b) $\begin{bmatrix} \phi_{HH,1} & \phi_{HL,1} \\ \phi_{LH,1} & \phi_{LL,1} \end{bmatrix} = \begin{bmatrix} 0.4 & 0.35 \\ 0.0 & 0.4 \end{bmatrix}$. Thus we have in (a) unidirectional causality from $x_H$ to $x_L$ and in (b) unidirectional causality from $x_L$ to $x_H$.

Since we assume stock sampling here, these causal patterns carry over to the corresponding MF- and LF-VAR processes under this parameterization. The innovations are mutually and serially independent standard normal $\mathbf{\eta}(\tau_L, k) \overset{i.i.d.}{\sim} \mathcal{N}(0_{2x1}, I_2)$. The full paper considers GARCH errors as well.

The low frequency sample size is fixed at $T_L = 100$ in this short paper. The full paper tries 50 and 500 as well. The ratio of sampling frequencies is taken from $m \in \{2, 3\}$, so the high frequency sample size is $T = mT_L \in \{200, 300\}$. The case that $(m, T_L) = (3, 100)$ can be thought of as a month versus quarter case covering 25 years. When $m$ takes a much larger value (e.g. $m = 12$ in month vs. year), our methodology loses practical applicability due to parameter proliferations. Handling a large $m$ is a future research question.

We aggregate the HF data $\{\mathbf{X}(\tau_L, k)\}$ into MF data $\{\mathbf{X}(\tau_L)\}^{T_L}$ and LF data $\{\mathbf{X}(\tau_L)\}^{T_L}$ using stock sampling. We then fit MF-VAR(1) and LF-VAR(1), which are correctly specified. Finally, we compute Wald statistics for two separate null hypotheses of high-to-low non-causality and low-to-high non-causality. The Wald statistic is computed via OLS with the Bartlett kernel HAC estimator (cfr. Newey and West (1987)). We choose a bandwidth based on Newey and West’s (1994) automatic bandwidth selection.

We circumvent size distortions using parametric bootstraps in Gonçalves and Killian (2004) with $N = 499$ replications. Gonçalves and Killian’s (2004) bootstrap does not require knowledge of the true error distribution and is robust to conditional heteroskedasticity of unknown form. See the full paper for a concrete procedure. Dufour, Pelletier, and Renault’s (2006) i.i.d. bootstrap is also considered there.

Simulation Results

Table 1 reports rejection frequencies. Note that, in case (a), size is computed with respect to low-to-high causality while power is computed with respect to high-to-low causality. In case (b), size is computed with respect to high-to-low causality, while power is computed with respect to low-to-high causality. Empirical size varies within $[0.041, 0.065]$, all close to the nominal size 5%. There are thus no serious size distortions in any case due to the bootstrap. Focusing on power, the results are consistent with the local power analysis. First, the MF causality test is slightly less powerful than LF causality test when high-to-low causality is concerned. For example, when $m = 2$, the MF high-to-low causality test has power 0.571 while LF high-to-low causality test has power 0.676. This is understandable since we know that working
on MF data and working LF data are equivalent in terms of high-to-low causal information, and the former involves more parameters. Second, the MF low-to-high causality test has clearly higher power than the LF counterpart. The difference is \(0.883 - 0.690 = 0.193\) for \(m = 2\) and \(0.832 - 0.310 = 0.522\) for \(m = 3\). This is consistent with our theoretical implication that working on MF data provides more low-to-high causal information than working on LF data does.

Table 1: Rejection Frequencies (Bivariate VAR with Stock Sampling)

Rejection frequencies at the 5% level for mixed and low frequency causality tests. The error term in the DGP is i.i.d. Stock sampling is used when we aggregate data. The two cases are (a) \(\phi_{HL,1} = 0\) and \(\phi_{LH,1} = 0.35\) (unidirectional high-to-low causality) and (b) \(\phi_{HL,1} = 0.35\) and \(\phi_{LH,1} = 0\) (unidirectional low-to-high causality). In case (a), size is computed with respect to low-to-high causality, while power is computed with respect to high-to-low causality. In case (b), size is computed with respect to high-to-low causality, while power is computed with respect to low-to-high causality. Newey and West’s (1987) HAC covariance estimator with Newey and West’s (1994) automatic bandwidth selection is used. Gonçalves and Killian’s (2004) wild bootstrap with \(N = 499\) replications is employed to avoid size distortions. \(m \in \{2, 3\}\) is the ratio of sampling frequencies and \(T_L = 100\) is the sample size in terms of low frequency.

<table>
<thead>
<tr>
<th>Size</th>
<th>Case (a): (x_H) causes (x_L)</th>
<th>Case (b): (x_L) causes (x_H)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(m=2)</td>
<td>(m=3)</td>
</tr>
<tr>
<td>MF</td>
<td>0.052</td>
<td>0.043</td>
</tr>
<tr>
<td>LF</td>
<td>0.041</td>
<td>0.049</td>
</tr>
<tr>
<td>Power</td>
<td>0.571</td>
<td>0.184</td>
</tr>
<tr>
<td>LF</td>
<td>0.676</td>
<td>0.274</td>
</tr>
</tbody>
</table>

7 Empirical Application

In this section we apply the mixed frequency causality test to U.S. macroeconomic data. We consider \(100\times\) annual log-differences of the U.S. monthly consumer price index for all items (CPI), monthly West Texas Intermediate spot oil price (OIL), and quarterly real GDP from July 1987 through June 2012. We use year-to-year growth rates to control for likely seasonality in each series.

Until here the present short paper had been discussing bivariate models only, but here we consider trivariate MF-VAR. The mixed frequency vector \(X(\tau_L)\) is now a \(7 \times 1\) vector \(X(\tau_L) = [CPI(\tau_L, 1), OIL(\tau_L, 1), CPI(\tau_L, 2), OIL(\tau_L, 2), CPI(\tau_L, 3), OIL(\tau_L, 3), GDP(\tau_L)]'\). In trivariate cases we need to take causality chains into account and hence iterate a MF-VAR model \(h \in \mathbb{N}\) times (cfr. Dufour and Renault (1998) and Dufour, Pelletier, and Renault (2006)). See the full paper for more details on trivariate cases.

Using mean-centered data, we fit an unrestricted MF-VAR(1) model with low frequency prediction horizon \(h \in \{1, \ldots, 5\}\) to monthly CPI, monthly OIL, and quarterly GDP. We therefore have two high frequency variables, one low frequency variable, \(m = 3\), \(T_L = 100\) quarters, and \(T = m \times T_L = 300\) months. Since the dimension of the MF-VAR is \(K = 7\), there are as many as 49 parameters even with the lag order one.

All six causal patterns (\(CPI \rightarrow OIL\), \(CPI \rightarrow GDP\), \(OIL \rightarrow GDP\) and their converses) are tested. We use Newey and West’s (1987) kernel-based HAC covariance estimator with Newey and West’s (1994) automatic lag selection. In order to avoid potential size distortions and to allow for conditional heteroskedasticity of unknown form, we use Gonçalves and Killian’s (2004) bootstrap with \(N = 999\) replications.

For the purpose of comparison, we also fit an unrestricted LF-VAR(4) model with low frequency prediction horizon \(h \in \{1, \ldots, 5\}\) to quarterly CPI, quarterly OIL, and quarterly GDP. Since parameter proliferation is less of an issue in LF-VAR, we let the lag order be 4 in order to take potential seasonality into account.
Table 2 presents bootstrapped p-values for MF and LF tests at each quarterly horizon. We denote whether rejection occurs at the 5% or 10% level. Note that the MF and LF tests reach very different conclusions. At the 5% level, for example, the MF case reveals three significant causal patterns: CPI causes GDP at horizon 3, OIL causes CPI at horizons 1 and 4, and GDP causes CPI at horizon 1. The LF case, however, has two different significant causal patterns: CPI causes OIL at horizon 1 and OIL causes GDP at horizons 2 and 4.

Note that significant causality from OIL to CPI is found by the MF approach but not by the LF approach, whether the 5% level or 10% level is used. Intuitively, such a causality should exist since (i) oil products are a component of the all-item CPI and (ii) crude oil is a key natural resource for most sectors. Our result suggests that the quarterly frequency is too coarse to capture the OIL-to-CPI causality while the mixed frequency data contain enough information for us to capture it successfully.

Table 2: Granger Causality Tests for CPI, OIL, and GDP

The mixed frequency approach uses monthly CPI, monthly OIL, and quarterly GDP. The low frequency approach uses all quarterly series. A box indicates rejection at the 5% level of the null hypothesis of non-causality at the quarterly horizon $h \in \{1, \ldots, 5\}$. A circle denotes rejection at the 10% level. The sample period covers July 1987 through June 2012, which has 300 months (100 quarters, 25 years). We use Newey and West’s (1987) kernel-based HAC covariance estimator with Newey and West’s (1994) automatic lag selection, and Gonçalves and Kilian’s (2004) bootstrapped p-value with $N = 999$ replications.

### Panel A. Mixed Frequency

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>0.391</td>
<td>0.128</td>
<td>0.559</td>
<td>0.636</td>
<td>0.165</td>
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<tr>
<td>CPI</td>
<td>0.195</td>
<td>0.098</td>
<td>0.049</td>
<td>0.100</td>
<td>0.180</td>
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<tr>
<td>CPI</td>
<td>0.680</td>
<td>0.548</td>
<td>0.236</td>
<td>0.300</td>
<td>0.196</td>
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<tr>
<td>OIL</td>
<td>0.002</td>
<td>0.182</td>
<td>0.439</td>
<td>0.029</td>
<td>0.605</td>
</tr>
<tr>
<td>OIL</td>
<td>0.015</td>
<td>0.570</td>
<td>0.583</td>
<td>0.125</td>
<td>0.500</td>
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<tr>
<td>GDP</td>
<td>0.724</td>
<td>0.833</td>
<td>0.895</td>
<td>0.855</td>
<td>0.946</td>
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</tbody>
</table>

### Panel B. Low Frequency

<table>
<thead>
<tr>
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<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI</td>
<td>0.035</td>
<td>0.095</td>
<td>0.095</td>
<td>0.116</td>
<td>0.492</td>
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<td>CPI</td>
<td>0.380</td>
<td>0.215</td>
<td>0.272</td>
<td>0.238</td>
<td>0.683</td>
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<td>OIL</td>
<td>0.145</td>
<td>0.044</td>
<td>0.088</td>
<td>0.027</td>
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<td>OIL</td>
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<td>0.320</td>
<td>0.986</td>
<td>0.710</td>
<td>0.521</td>
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<tr>
<td>GDP</td>
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<td>0.497</td>
<td>0.323</td>
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<td>0.645</td>
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<tr>
<td>GDP</td>
<td>0.095</td>
<td>0.164</td>
<td>0.516</td>
<td>0.376</td>
<td>0.541</td>
</tr>
</tbody>
</table>

8 Concluding Remarks

Time series processes are often sampled at different frequencies and are typically aggregated to the common lowest frequency to test for Granger causality. This paper compares testing for Granger causality with all series aggregated to the common lowest frequency, and testing for Granger causality taking advantage of all the series sampled at whatever frequency they are available. We rely on mixed frequency vector autoregressive models to implement the new class of Granger causality tests.

We show that mixed frequency causality tests better recover causality patterns in an underlying high frequency process than the traditional low frequency approach. Moreover, we show formally that mixed frequency causality tests have higher asymptotic power against local alternatives and show via simulation that this also
holds in finite samples involving realistic data generating processes. The simulations indicate that the mixed frequency approach works well for small differences in sampling frequencies like month vs. quarter.

We apply the mixed frequency causality test to a monthly consumer price index, monthly crude oil prices, and the real GDP in the U.S. We also apply the conventional low frequency causality test to the aggregated quarterly price series and the real GDP for comparison. These two approaches produce very different results, and the mixed frequency approach yields more intuitive results.

References


