Bootstrap Inference for Instrumental Variable Models

with Many Weak Instruments

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In this paper, we study the asymptotic validity of bootstrap procedures for the limited information maximum likelihood (LIML) estimator when the instruments in instrumental variable (IV) regression may be weak and the number of instruments goes to infinity with the sample size. We show analytically that a standard residual-based bootstrap procedure cannot consistently estimate the limiting distribution of the LIML estimator. The foremost reason is that it fails to well mimic the parameter that characterizes the identification strength in the sample. It also fails to capture some important properties of the disturbances. We also consider the restricted efficient (RE) bootstrap procedure of Davidson and MacKinnon (2008, 2010) which generates bootstrap data under the null (Restricted) and uses an efficient estimator of the coefficient of the reduced-form equation (Efficient). Similarly, we show that the RE bootstrap procedure is also first order invalid under many/many weak instruments, even if it is able to well mimic more parameters in the distribution of interest than the standard procedure and has therefore relatively smaller size distortion in finite sample. Finally, we propose a modified bootstrap procedure which consistently estimates the limiting distribution of the LIML estimator. The modified procedure achieves this goal by using properly re-scaled residuals and by using a consistent estimator of the parameter that captures the identification strength. A Monte Carlo experiment shows that confidence intervals based on Hansen, Hausman and Newey (2008)'s normal approximation can be very distorted in finite sample, especially when the degree of endogeneity is high. Instead, our modified bootstrap procedure greatly reduces these distortions.