Age-Period-Cohort Decomposition Using Principal Components or Partial Least Squares: A Simulation Study

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Abstract: Age-period-cohort decomposition requires an identification assumption because there is a linear relationship among age, survey period, and birth cohort (age + cohort = period). This paper proposes new decomposition methods based on factor models such as principal components model and partial least squares model. Although factor models have been applied to overcome the problem of many observed variables with possible co-linearity, they are applied to overcome the perfect co-linearity among age, period, and cohort dummy variables. Since any unobserved factor in the factor model is represented as a linear combination of the observed variables, the parameter estimates for age, period, and cohort effects are automatically obtained after the application of these factor models. Simulation results suggest that in most cases, the performance of the proposed method is at least comparable to conventional methods but it has a model-selection problem.

Keywords: Age-period-cohort decomposition; Identification problem; Partial least squares; Principal components

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1. Introduction

Classified by age and period, repeated cross-section data such as those obtained in a consumer survey, are compiled and expected to provide useful information using the following model:

$$y_{ij} = \alpha + A_i + P_j + C_k + \varepsilon_{ij}, \quad i = 1,2,...,I; \ j = 1,2,...,J; \ k = 1,2,...,K. \qquad (1)$$

Here, α is the constant term; A_i, the effect of the age group; P_j, the effect of the survey period; C_k, the effect of the birth cohort; and ε_{ij} a normal disturbance term with mean zero and variance σ^2. In the present paper, this type of data is referred to as cohort data, and the model (1) is referred to as cohort model. Furthermore, it is assumed that the range of the age group coincides with the interval of the survey period. Thus, it can be shown that $K = I + J - 1$ and $k = I - i + j$. Without loss of generality, the parameters A_i, P_j, and C_k in (1) are subject to the following constraint.

$$\sum_{i=1}^{I} A_i = \sum_{j=1}^{J} P_j = \sum_{k=1}^{K} C_k = 0. \qquad (2)$$

Model (1) with constraint (2) can be rewritten in vector and matrix notations as

$$y = X\beta + \varepsilon, \qquad (3)$$

where y is $IJ \times 1$ vector, X is $IJ \times [2(I+J)-3]$ design matrix, β is $[2(I+J)-3] \times 1$ vector specified as $\beta = (\alpha, A_1,..., A_{I-1}, P_1,..., P_{J-1}, C_1,..., C_{K-1})$, and ε is $IJ \times 1$ vector. Generally, the maximum likelihood procedure is applied to obtain the estimate of the parameter vector β in (3), but in the cohort model, the estimates cannot be uniquely identified because the effect parameters are linearly dependent, for example, age + cohort = period. Hence, additional information on the parameters is required in order to completely specify this model.

2. Conventional Methods

2.1 Deaton-Paxson (1994) method

In economics literature, since the empirical work by Deaton and Paxson (1994), it has been assumed that the period effect is orthogonal to a linear time trend and average zero. In other words, Deaton and Paxson assume that the period effect is identical to a business cycle effect and that all trend movements are caused by the age and cohort effects. Thus, the following identification equations are obtained.

$$P_{J-1} = \sum_{j=1}^{J-2} jP_j - J\sum_{j=1}^{J-2} P_j, \quad P_J = (J-1)\sum_{j=1}^{J-2} P_j - \sum_{j=1}^{J-2} jP_j. \qquad (4)$$

In the present paper, the identification method based on (4) is called as the DP method. The DP method has been widely applied to empirical studies (Attanasio 1998; Jianakoplos and Bernasek 2006; Kalwij and Alessie 2007).

2.2 Nakamura's (1982, 1986) Bayesian cohort model

In Nakamura's (1982, 1986) Bayesian cohort (BC) model, smoothness prior information is introduced for identification. It is assumed that the parameters of age, period, and cohort effects change gradually. In other words, the first-order differences in the successive effect parameters, $A_i - A_{i+1}$, $P_j - P_{j+1}$, and $C_k - C_{k+1}$ are close to zero for $i = 1,...,I-1$; $j = 1,...,J-1$ and $k = 1,...,K-1$. Therefore, the following objective function is obtained.

$$\frac{1}{\sigma_A^2}\sum_{i=1}^{I-1}(A_i - A_{i+1})^2 + \frac{1}{\sigma_P^2}\sum_{j=1}^{J-1}(P_j - P_{j+1})^2 + \frac{1}{\sigma_C^2}\sum_{k=1}^{K-1}(C_k - C_{k+1})^2 \rightarrow \min, \quad (5)$$

where σ_A^2 , σ_P^2 , and σ_C^2 are properly chosen constants and are called the hyperparameters in Bayesian modeling. Assuming that the effect parameters have prior normal distribution, Nakamura estimates β by the mode of the posterior density proportional to $f(y \mid \beta, \sigma^2) \bullet \pi(\beta \mid \sigma_A^2, \sigma_P^2, \sigma_C^2)$, where $f(y \mid \beta, \sigma^2)$ is the likelihood function for the overall model fitness, and $\pi(\beta \mid \sigma_A^2, \sigma_P^2, \sigma_C^2)$ is the prior distribution function for the smoothness condition. In order to determine the values of the hyperparameters, Nakamura adopts the Akaike Bayesian Information Criterion (ABIC), proposed by Akaike (1980). The ABIC is defined by

$$\text{ABIC} = -2\ln\int f(y \mid \beta, \sigma^2) \bullet \pi(\beta \mid \sigma_A^2, \sigma_P^2, \sigma_C^2) d\beta + 2h, \quad (6)$$

where h is the number of hyperparameters. In model selection, the model with the smaller ABIC is selected.

3. Proposed Methods

3.1 Principal components regression

The proposed identification methods are based on factor methods, such as the principal components (PC) regression and partial least squares (PLS) regression, and they involve a two-step procedure. In the first step, the factor methods are applied to the data sets of y and X to obtain the unobserved factors. Next, the estimates $\hat{\beta}$ in (3) are automatically obtained, because any unobserved factor is represented as a linear combination of the observed components of X. Factor models have been applied to overcome the problem of many observed variables with possible co-linearity. In the present study, however, factor models are applied to overcome the identification problem of the cohort model. Accordingly, I reconsider regression (3) with $N = IJ$ and $M = 2(I+J)-3$ as follows.

First, the PC regression is considered. The data are normalized to obtain zero mean and unit variance before the application of the PC regressions. Hence,

$$y_n = \beta' x_n + \varepsilon_n; \quad n = 1,...,N, \tag{7}$$

where $\beta = (\beta_1 \cdots \beta_M)'$, and $x_n = (x_{1n} \cdots x_{Mn})'$. I assume that the co-movements in x_n can be captured by $r \times 1$ vector of unobserved factors F_n, i.e.,

$$x_n = \Lambda' F_n + e_n, \tag{8}$$

where Λ is an $r \times M$ matrix of parameters, generally called the factor loadings matrix, that indicates the relation of the individual components in x_n to each of the r factors. In (8), e_n is composed of a zero-mean $I(0)$ vector of disturbance terms. r denotes the small number of linear combinations of x_n that represent the factors and act as the predictors for y_n. In the PC method, given the value of r, the estimates of Λ and F_n are obtained by solving

$$\frac{1}{NM}\sum_{i=1}^{M}\sum_{j}^{N}(x_{ij} - \lambda_i' F_n)^2 \to \text{Min} \tag{9}$$

where λ_i is an $r \times 1$ vector of loadings that represent M columns of $\Lambda = (\lambda_1 \cdots \lambda_M)$. The solution for (9) can be determined by using the eigenvectors corresponding to the r largest eigenvalues of the second moment matrix $X'X$. In the cohort model (3), $\hat{\beta}$ can be obtained after the above PC regression.

Another problem to be resolved is the determination of the value of r. However, this problem has hitherto not been resolved in a statistically relevant framework. Bai and Ng (2002) proposed an asymptotic theory for determining the number of factors under the framework of large cross-sections and large time dimensions. However, their proposed criteria cannot be applied in the present paper, because normality is not a reasonable distributional approximation for the data of X (Kolenikov and Angeles 2009). Since the rank of the matrix X is $M-1$, $M-1$ cases are considered for r ($r = 1,2,...,M-1$). Similar discussions are applicable to the PLS method.

3.2 Partial least squares

The basic concept of PLS regression is similar to the PC regression in that any factor is obtained as a linear combination of the original explanatory variables and is adopted as a regressor. The main difference between the PC and PLS regressions is that the factors in the PC regression are constructed considering only independent variables, whereas the PLS factors are obtained considering the relationship between dependent and independent variables. It is assumed for simplicity that all variables are normalized to have zero mean and that there are k PLS factors. Helland (1988, 1990) provided the following simple algorithm.

Step1: Set $u_n = y_n$ and $v_{in} = x_{in}$, $i = 1,...,M$. Set $j = 1$.

Step2: Obtain the j-th PLS factor as $f_{jn} = w_j' v_n$, where $w_j (w_{1j} \cdots w_{Mj})'$, $w_{ij} = Cov(u_n, v_{in}),\ i = 1,...,M$.

Step3: Regress u_n and v_{in}, $i = 1,...,M$, on f_{jn}. Denote the residuals of these regressions by $\tilde{u}_n$ and $\tilde{v}_{in}$, respectively.

Step4: If $j = k$, then stop; else, set $u_n = \tilde{u}_n$, $v_{in} = \tilde{v}_{in}$, $i = 1,...,M$, and $j = j+1$ and go to Step2.

In Step3, the following equation is obtained for $j = k$.

$$y_n = \{\hat{\phi}_1 w_1 + \hat{\phi}_2 w_2 (I - \hat{\theta}_1 w_1) + \cdots + \hat{\phi}_k w_k (I - \hat{\theta}_1 w_1) \cdots (I - \hat{\theta}_{k-1} w_{k-1})\} x_n + \tilde{u}_n, \quad (10)$$

where $\hat{\phi}_j$ is the parameter obtained by regressing u_n on f_{jn} and $\hat{\theta}_j$ is the parameter vector obtained by regressing v_{in} on f_{jn}. As in the PC regression, determining the value of k has not been resolved in a statistically relevant framework. Furthermore, the number of iterations need to obtain the convergence in the above algorithm cannot be predetermined. Thus, $M/4$ cases are arbitrarily considered for k ($k = 1,2,...,M/4$).

4. Simulation Study

4.1 Comparisons among the DP, PC, and PLS methods

The following three data generating processes (DGPs) are considered for each effect.

DGP1: $z_l = 0,$

DGP2: $z_l = \eta_l,$

DGP3: $z_l = z_{l-1} + \eta_l,$

DGP4: $z_l = 2z_{l-1} - z_{l-2} + \eta_l,$

where $\eta_l \sim i.i.d.N(0,1)$, $l = 1,..,L.$ and $z_{-1} = z_0 = 0.$ It is generally difficult to consider typical DGPs for age, period and cohort effects. Since the seminal work by Nelson and Plosser (1982), the time series properties of economic variables have been examined in numerous empirical studies. DGP2 is the simplest process for stationary time series. DGP3 is the so-called random walk process and is usually adopted for financial time series such as foreign exchange rates. DGP4 is suitable for modeling stock variables such as monetary supply. Thus, it is logical to consider that the three DGPs mentioned above are typical of economic time series. Selecting one DGP from the four alternatives for each effect produces 64 data types for cohort data. The simulations are performed as follows. First, I select one DGP from the four alternatives for each effect, and generate η_l artificially. Then, each effect is standardized with zero mean and unit variance. Next, cohort data are obtained by combining the three effects and adding i.i.d. normal noise with unit variance. Finally, the three identification methods (DP, PC, and PLS) are applied to artificial cohort data. Then, the difference between the estimated parameter value and the assumed parameter value is calculated for the age, period, and cohort effects, and the overall root mean square error (RMSE) is obtained by adding the squared differences for individual effects. This simulation is replicated 100 times with 10 age groups and 10 periods, and the average RMSE is obtained for each method.

Table 1 presents the simulation results. As mentioned in the previous section, determining the value of r in the PC regression and the value of k in the PLS regression are open questions. In the case of the PC regression, the following three cases are considered.

PC1: The PC regression with only the largest eigenvalues,

PCm: The PC regression with $M-1$ nonzero eigenvalues,

PCs: The PC regression providing the smallest RMSE.

PCm is substantially identical to the method proposed by Yang et al. (2004), Yang (2008) and Yang et al. (2008). In this simulation study $M=36$. With regard to the PLS regression, similar alternatives are considered; however, PLSm indicates the PLS regression with $k=M/4$. I consider the simulation result for the case wherein all of the effects are artificially generated from DGP1, denoted by (1 1 1). The corresponding average RMSEs are 0.449 (DP), 0.032 (PC1), 0.430 (PCm), 0.032 (PCs), 0.206 (PLS1), 0.246 (PLSm), and 0.206 (PLSs). $r=1$ for the PCs model and $k=1$ for the PLSs model were obtained.

I first consider the simulation results obtained by the DP method. The performances are very poor when the period effect is generated from DGP3 or DGP4. Considering the assumption in the DP method, these results are straightforward. Next, I consider the simulation results obtained by the PC method. The performances of the PCs method are considerably better than those of the DP method. Furthermore, the number of selected components in the PCs method ranges from 1 to 36 but is larger than 18 except for two data types. In most cases, there is no considerable difference in RMSE between the PCm and PCs methods, and therefore only the PCm method is considered in subsequent studies. Similar results are obtained for the PLS method, and the performances of the PLSs method are worse than those of the PCs method. Thus, in the next simulation study, only the PCm method is considered.

4.2 Comparisons between the PCm and BC methods

The purpose of this simulation is to examine frequency count of selecting the correct model from eight alternatives: APC, AP, AC, PC, A, P, C, and no effect models. Considered DGPs are completely the same as those in the previous section. Regarding the BC method, the model fitness of eight alternatives can be uniformly evaluated using the information criterion, ABIC. In the framework of the PCm method, the following mix procedure is adopted: the PCm regression is applied to the APC model and the conventional regression is applied to the other seven models. Furthermore, no unique criterion can be expected for the PCm method, and the following three alternative information criteria are adopted: the Akaike information criterion (AIC), the Schwarz Bayesian information criterion (BIC), and Hannan and Quinn criterion (HQC).

Table 2 present selected results regarding the frequency count of the selected model. The boldface letter indicates the frequency count of the correct selection. This table suggests that any method does not always beat the other three methods. However, it can be concluded that the BC method outperforms the other three methods in most cases. Table 3 present RMSE for each effect obtained by each method for 40 data types. It is clearly shown that the BC method outperforms the PCm method except when the true DGP contains neither age or period or cohort effect. In other words, if the cohort data are generated from the full model, the performances of the PCm method are better than those of the BC method. For example, consider the simulation result for the case where the cohort data are generated from the process (2 4 2). The RMSEs obtained by the PCm method for age, period, or cohort effects are respectively 0.326, 0.347, and 0.596, and those obtained by the BC method are respectively 0.791, 0.766, and 1.450.

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Table 1. Simulation results regarding RMSE											
DGP			DP	Principal components regression				Partial least squares			
Age	Period	Cohort		PC1	PCm	PCs	r	PL1	PLm	PLs	k
1	1	1	0.449	0.032	0.430	0.032	1	0.206	0.246	0.206	1
1	1	2	0.456	0.679	0.464	0.464	36	0.623	0.505	0.505	10
1	1	3	0.459	0.672	0.611	0.607	35	0.661	0.619	0.619	9
1	1	4	0.456	0.665	0.686	0.617	6	0.676	0.662	0.648	2
1	2	1	0.666	0.471	0.438	0.295	19	0.415	0.414	0.386	3
1	2	2	0.691	0.823	0.480	0.480	36	0.708	0.581	0.581	10
1	2	3	0.679	0.822	0.608	0.606	35	0.765	0.691	0.691	10
1	2	4	0.697	0.815	0.710	0.662	19	0.782	0.744	0.732	2
1	3	1	1.018	0.461	0.446	0.300	19	0.391	0.425	0.391	1
1	3	2	1.100	0.824	0.496	0.480	35	0.715	0.589	0.587	4
1	3	3	1.098	0.815	0.609	0.609	36	0.761	0.678	0.678	9
1	3	4	1.070	0.806	0.686	0.662	21	0.769	0.728	0.723	2
1	4	1	1.377	0.447	0.463	0.305	21	0.369	0.433	0.369	1
1	4	2	1.352	0.813	0.475	0.468	35	0.722	0.578	0.577	6
1	4	3	1.346	0.805	0.596	0.596	36	0.746	0.663	0.663	11
1	4	4	1.343	0.804	0.714	0.678	19	0.762	0.718	0.717	8
2	2	1	0.668	0.671	0.444	0.337	25	0.561	0.577	0.553	3
2	2	2	0.651	0.953	0.474	0.468	35	0.815	0.742	0.742	7
2	2	3	0.636	0.950	0.607	0.607	36	0.850	0.742	0.742	14
2	2	4	0.677	0.943	0.694	0.675	25	0.872	0.815	0.814	6
2	3	1	1.100	0.663	0.433	0.332	26	0.550	0.551	0.536	4
2	3	2	1.075	0.951	0.483	0.483	36	0.820	0.719	0.719	6
2	3	3	1.069	0.947	0.631	0.630	35	0.855	0.794	0.793	7
2	3	4	1.118	0.944	0.712	0.690	26	0.867	0.829	0.828	7
2	4	1	1.344	0.656	0.467	0.350	23	0.542	0.559	0.540	4
2	4	2	1.358	0.940	0.480	0.480	36	0.819	0.764	0.764	19
2	4	3	1.347	0.936	0.588	0.588	36	0.845	0.748	0.747	7
2	4	4	1.400	0.934	0.702	0.677	23	0.853	0.806	0.802	4
3	3	1	1.108	0.665	0.470	0.367	23	0.553	0.581	0.553	1
3	3	2	1.091	0.952	0.485	0.485	36	0.826	0.749	0.748	6
3	3	3	1.058	0.945	0.618	0.618	36	0.855	0.756	0.756	12
3	3	4	1.038	0.941	0.699	0.679	28	0.852	0.797	0.797	9
3	4	1	1.365	0.655	0.451	0.360	25	0.530	0.564	0.530	1
3	4	2	1.392	0.944	0.484	0.484	36	0.823	0.789	0.782	4
3	4	3	1.344	0.942	0.652	0.644	35	0.846	0.813	0.812	7
3	4	4	1.366	0.933	0.695	0.674	25	0.848	0.797	0.793	4
4	4	1	1.368	0.659	0.481	0.392	23	0.523	0.584	0.523	1
4	4	2	1.370	0.941	0.504	0.495	35	0.809	0.776	0.760	4
4	4	3	1.330	0.944	0.610	0.610	36	0.831	0.756	0.753	4
4	4	4	1.404	0.932	0.730	0.714	23	0.845	0.811	0.809	4

Table 2. Simulation results regarding model selections

DGP			Methods	Frequency count of the selected model							
Age	Period	Cohort		APC	AP	AC	PC	A	P	C	No
1	1	1	PC-AIC	0.02	0.01	0.01	0.00	0.04	0.05	0.03	**0.84**
1	1	1	PC-HQC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**
1	1	1	PC-BIC	0.00	0.00	0.00	0.00	0.00	0.00	0.00	**1.00**
1	1	1	BC	0.00	0.01	0.00	0.00	0.05	0.05	0.01	**0.88**
1	1	4	PC-AIC	0.02	0.19	0.05	0.08	0.00	0.01	**0.65**	0.00
1	1	4	PC-HQC	0.00	0.12	0.00	0.01	0.06	0.05	**0.61**	0.15
1	1	4	PC-BIC	0.00	0.01	0.00	0.00	0.04	0.02	**0.13**	0.80
1	1	4	BC	0.00	0.05	0.08	0.08	0.00	0.00	**0.79**	0.00
1	2	4	PC-AIC	0.14	0.46	0.00	**0.39**	0.00	0.01	0.00	0.00
1	2	4	PC-HQC	0.02	0.54	0.00	**0.23**	0.00	0.21	0.00	0.00
1	2	4	PC-BIC	0.00	0.29	0.00	**0.02**	0.00	0.63	0.00	0.06
1	2	4	BC	0.07	0.39	0.00	**0.54**	0.00	0.00	0.00	0.00
2	2	1	PC-AIC	0.11	**0.89**	0.00	0.00	0.00	0.00	0.00	0.00
2	2	1	PC-HQC	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00
2	2	1	PC-BIC	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00
2	2	1	BC	0.09	**0.91**	0.00	0.00	0.00	0.00	0.00	0.00
2	2	4	PC-AIC	**0.44**	0.56	0.00	0.00	0.00	0.00	0.00	0.00
2	2	4	PC-HQC	**0.21**	0.79	0.00	0.00	0.00	0.00	0.00	0.00
2	2	4	PC-BIC	**0.03**	0.92	0.00	0.00	0.02	0.01	0.00	0.02
2	2	4	BC	**0.45**	0.55	0.00	0.00	0.00	0.00	0.00	0.00
2	3	1	PC-AIC	0.07	**0.93**	0.00	0.00	0.00	0.00	0.00	0.00
2	3	1	PC-HQC	0.01	**0.99**	0.00	0.00	0.00	0.00	0.00	0.00
2	3	1	PC-BIC	0.00	**1.00**	0.00	0.00	0.00	0.00	0.00	0.00
2	3	1	BC	0.04	**0.96**	0.00	0.00	0.00	0.00	0.00	0.00
2	3	4	PC-AIC	**0.34**	0.64	0.02	0.00	0.00	0.00	0.00	0.00
2	3	4	PC-HQC	**0.11**	0.82	0.01	0.00	0.06	0.00	0.00	0.00
2	3	4	PC-BIC	**0.00**	0.84	0.01	0.00	0.13	0.01	0.00	0.01
2	3	4	BC	**0.43**	0.56	0.01	0.00	0.00	0.00	0.00	0.00
3	3	3	PC-AIC	**0.82**	0.13	0.02	0.03	0.00	0.00	0.00	0.00
3	3	3	PC-HQC	**0.42**	0.41	0.06	0.07	0.01	0.00	0.03	0.00
3	3	3	PC-BIC	**0.11**	0.59	0.05	0.04	0.05	0.05	0.04	0.07
3	3	3	BC	**0.73**	0.16	0.05	0.05	0.00	0.00	0.01	0.00
3	3	4	PC-AIC	**0.40**	0.58	0.01	0.00	0.00	0.00	0.01	0.00
3	3	4	PC-HQC	**0.20**	0.77	0.00	0.00	0.02	0.00	0.00	0.01
3	3	4	PC-BIC	**0.05**	0.70	0.00	0.00	0.08	0.08	0.01	0.08
3	3	4	BC	**0.45**	0.52	0.00	0.01	0.01	0.00	0.01	0.00
3	4	1	PC-AIC	0.06	**0.86**	0.07	0.01	0.00	0.00	0.00	0.00
3	4	1	PC-HQC	0.00	**0.97**	0.02	0.01	0.00	0.00	0.00	0.00
3	4	1	PC-BIC	0.00	**0.99**	0.01	0.00	0.00	0.00	0.00	0.00
3	4	1	BC	0.02	**0.91**	0.04	0.01	0.00	0.00	0.02	0.00
3	4	4	PC-AIC	**0.23**	0.55	0.17	0.02	0.02	0.00	0.01	0.00
3	4	4	PC-HQC	**0.10**	0.64	0.10	0.01	0.09	0.01	0.03	0.02
3	4	4	PC-BIC	**0.02**	0.58	0.02	0.00	0.23	0.05	0.02	0.08
3	4	4	BC	**0.35**	0.50	0.10	0.03	0.00	0.00	0.02	0.00
4	4	2	PC-AIC	**0.40**	0.00	0.27	0.25	0.00	0.00	0.08	0.00
4	4	2	PC-HQC	**0.19**	0.00	0.35	0.25	0.00	0.00	0.21	0.00
4	4	2	PC-BIC	**0.04**	0.01	0.36	0.26	0.01	0.00	0.32	0.00
4	4	2	BC	**0.32**	0.00	0.28	0.27	0.00	0.00	0.13	0.00

Note: The boldface letter indicates the frequency count of the correct selection.

Table 3. Simulation results regarding RMSE

DGP				PCm				BC		
Age	Period	Cohort		Age	Period	Cohort		Age	Period	Cohort
1	1	1		0.308	0.305	0.530		0.089	0.089	0.085
1	1	2		0.324	0.331	0.573		0.086	0.077	0.502
1	1	3		0.416	0.427	0.762		0.089	0.094	0.399
1	1	4		0.463	0.456	0.862		0.186	0.166	0.415
1	2	1		0.308	0.305	0.543		0.077	0.304	0.103
1	2	2		0.322	0.334	0.600		0.087	0.325	0.534
1	2	3		0.426	0.403	0.760		0.130	0.329	0.439
1	2	4		0.491	0.470	0.889		0.345	0.440	0.686
1	3	1		0.310	0.311	0.554		0.075	0.281	0.110
1	3	2		0.327	0.318	0.628		0.101	0.304	0.557
1	3	3		0.418	0.409	0.763		0.185	0.339	0.518
1	3	4		0.474	0.470	0.854		0.376	0.458	0.741
1	4	1		0.321	0.317	0.577		0.081	0.237	0.116
1	4	2		0.327	0.319	0.594		0.332	0.402	0.781
1	4	3		0.403	0.419	0.743		0.268	0.356	0.621
1	4	4		0.488	0.483	0.894		0.337	0.434	0.701
2	2	1		0.311	0.312	0.549		0.299	0.301	0.106
2	2	2		0.317	0.335	0.591		0.354	0.359	0.616
2	2	3		0.424	0.423	0.754		0.420	0.428	0.668
2	2	4		0.460	0.475	0.871		0.522	0.532	0.856
2	3	1		0.311	0.316	0.531		0.296	0.272	0.108
2	3	2		0.320	0.337	0.604		0.474	0.482	0.875
2	3	3		0.416	0.439	0.791		0.467	0.491	0.778
2	3	4		0.473	0.487	0.892		0.525	0.523	0.879
2	4	1		0.327	0.328	0.578		0.360	0.308	0.364
2	4	2		0.326	0.347	0.596		0.791	0.766	1.450
2	4	3		0.414	0.398	0.731		0.597	0.571	1.066
2	4	4		0.479	0.460	0.883		0.619	0.581	1.055
3	3	1		0.331	0.325	0.584		0.284	0.287	0.143
3	3	2		0.321	0.343	0.606		0.582	0.599	1.108
3	3	3		0.421	0.440	0.767		0.490	0.489	0.858
3	3	4		0.480	0.484	0.871		0.541	0.537	0.903
3	4	1		0.315	0.323	0.558		0.341	0.310	0.393
3	4	2		0.341	0.341	0.599		0.743	0.741	1.407
3	4	3		0.452	0.451	0.812		0.657	0.642	1.134
3	4	4		0.467	0.502	0.863		0.582	0.595	1.004
4	4	1		0.340	0.337	0.595		0.417	0.391	0.597
4	4	2		0.345	0.339	0.631		0.767	0.770	1.486
4	4	3		0.406	0.420	0.765		0.644	0.656	1.206
4	4	4		0.490	0.496	0.915		0.635	0.632	1.141